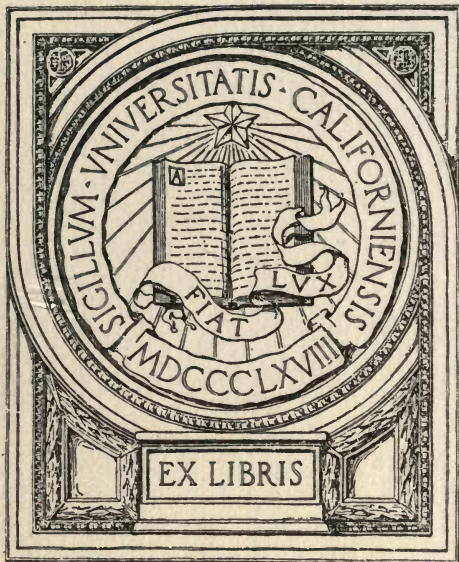


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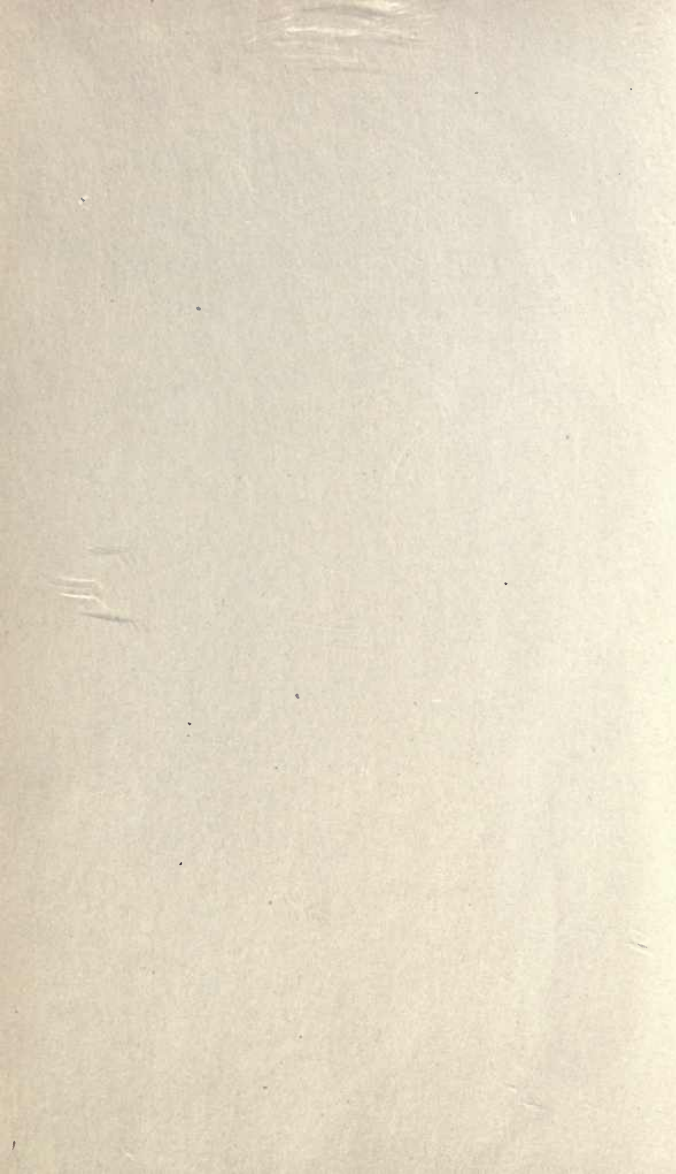


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C.G.S. SYSTEM OF UNITS.



ILLUSTRATIONS
OF THE
C.G.S. SYSTEM OF UNITS
WITH
TABLES OF PHYSICAL CONSTANTS.

BY
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Fourth Edition, 1891.



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PREFACE.

THIS work, in its original form, was published by the Physical Society of London, in 1875, as "Illustrations of the C.G.S. System of Units."

A greatly enlarged edition was issued in 1879 by the present publishers, and the title was changed to "Units and Physical Constants," the letters "C.G.S." being suppressed from an idea that their strangeness would unfavourably influence the sale. This new title was retained in the "second edition" issued in 1886.

The C.G.S. units having now become the accepted standards of reference throughout the scientific world, there is no longer any reason for suppressing the name. In the present edition they are accordingly restored to their place in the title, which is thus brought into closer agreement with the contents of the book.

The German edition, which was announced in our last preface as about to appear, was published at Leipsic in 1888 by Ambrosius Barth. It is a close translation as regards the portions relating to units, but contains large changes in the experimental data, some years having been devoted to this portion of the undertaking after the original manuscript had been completed. The present edition has been closely compared with the German

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rendering, and in several instances I have been glad to avail myself of the permission of the editors (Drs. Chappuis and Kreichgauer) to make use of their material.

A Russian translation was published at St. Petersburg in 1888, edited by Mr. P. N. Verbitsky and Captain J. Th. Gerabiatieff, and revised by Professor S. A. Ousoff. I am indebted to these gentlemen for pointing out several errors which they discovered in a searching scrutiny not only of the book itself but of the authorities quoted in it. They have, for example, corrected some numbers quoted from Péclet, by the aid of a table of errata in Péclet's book which I had overlooked.

The following are the principal items of new matter :—

1. A collection of determinations of viscosities of liquids and gases; in connection with which I have to express my deep obligations to Mr. Carl Barus, who lent me a large bundle of MSS., including all recent results on this subject.
2. A summary of recent investigations of the magnetic properties of iron and other substances, with the necessary expositions of theory and terminology. Under this head I have to acknowledge very special obligations to Mr. Shelford Bidwell, some six or eight pages of the new matter being substantially from his pen. I have also received valuable aid from Professor Ewing and Dr. Hopkinson.
3. A revolutionising of the introductory sections on Heat, rendered necessary by Rowland's discovery that the specific heat of water decreases by 1 per

cent. as the temperature rises from 5° to 30° C. The new sections include Rowland's results for the mechanical equivalent of heat, and for the comparison of the thermodynamic scale of temperature with the air thermometer and with a Kew standard.

4. Two sections on self-induction, mutual induction, and the theory and terminology of alternating currents.
5. Two sections on the dimensions of electrical and magnetic quantities in terms of K and μ . Under this head I have to acknowledge kind assistance from Professors Rücker and Fitzgerald.
6. A table of values of the magnetic elements at a number of stations in the British Isles, selected from the results of the recent survey of Professors Rücker and Thorpe.

Information on several subjects has been brought down to date ; including

Wave-lengths of light.

Mercury standards of resistance, and their correction for temperature.

Standards of light-giving power.

Emission of heat.

Conduction of heat by liquids.

Departure from Boyle's law at very high pressures.

Compression of fresh water, sea water, mercury, and glass, at very high pressures.

Coefficients of diffusion of liquids and gases, with a simplified exposition.

Comparison of the mass of unit volume of water with standards of mass.

Besides the names above mentioned, I desire to return my thanks to several well-known men of science who have aided me by corrections or suggestions, all of which have been carefully considered, though in some instances I have not seen my way to carry them out.

PREFACE TO "ILLUSTRATIONS OF THE C.G.S.
SYSTEM OF UNITS," 1875.

THE quantitative study of physics, and especially of the relations between different branches of physics, is every day receiving more attention.

To facilitate this study, by exemplifying the use of a system of units fitted for placing such relations in the clearest light, is the main object of the present treatise.

A complete account is given of the theory of units *ab initio*. The Centimetre-Gramme-Second (or C.G.S.) system is then explained; and the remainder of the work is occupied with illustrations of its application to various branches of physics. As a means to this end, the most important experimental data relating to each subject are concisely presented on *one uniform scale*—a luxury hitherto unknown to the scientific calculator.

I am indebted to several friends for assistance in special departments—but especially to Professor Clerk Maxwell and Professor G. C. Foster, who revised the entire manuscript of the work in its original form.

Great pains have been taken to make the work correct as a book of reference. Readers who may discover any errors will greatly oblige me by pointing them out.

EXTRACT FROM PREFACE TO FIRST EDITION OF
“UNITS AND PHYSICAL CONSTANTS,” 1879.

THIS Book is substantially a new edition of my “Illustrations of the C.G.S. System of Units,” published in 1875 by the Physical Society of London, supplemented by an extensive collection of physical data. The title has been changed with the view of rendering it more generally intelligible.

Additional explanations have been given upon some points of theory, especially in connection with Stress and Strain, and with Coefficients of Diffusion. Under the former head, I have ventured to introduce the terms “resilience” and “coefficient of resilience,” in order to avoid the multiplicity of meanings which have become attached to the word “elasticity.”

A still greater innovation has been introduced in an extended use of the symbols and processes of multiplication and division, in connection with equations which express not numerical but physical equality. The advantages of this mode of procedure are illustrated by its application to the solution of the most difficult problems on units that I have been able to collect from standard text-books.

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A Dutch translation of the first edition of this work has been made by Dr. C. J. MATTHES, Secretary of the Royal Academy of Sciences of Amsterdam, and was published in that city in 1877.

EXTRACT FROM PREFACE TO THE EDITION OF 1886.

IN collecting materials for this edition, I have gone carefully through the Transactions and Proceedings of the Royal Society, the Royal Society of Edinburgh, and the Physical Society of London, from 1879 onwards, besides consulting numerous papers, both English and foreign, which have been sent to me by their authors. I have also had the advantage of the co-operation of Dr. Pierre Chappuis (of the *Bureau International des Poids et Mesures*), who has for some years been engaged in preparing

a German edition. Several items have been extracted from the very elaborate and valuable *Physikalisch-Chemische Tabellen* of Landolt and Börnstein (Julius Springer, Berlin, 1883).

A Supplemental Section has been added on physical deductions from the dimensions of units ; a simplification has been introduced in the discussion of adiabatic compression ; and the account of thermoelectricity has been rewritten and enlarged. The name "thermoelectric height" has been introduced to denote the element usually represented by the ordinates of a thermoelectric diagram.

The adoption of the Centimetre, Gramme, and Second, as the fundamental units by the International Congress of Electricians at Paris in 1881, led to the immediate execution of a French translation of this work, which was published at Paris by Gauthier-Villars in 1883. The German translation was commenced about the same time, but the desire to perfect its collection of physical data has caused much delay. It will be brought out by Ambrosius Barth, the publisher of Wiedemann's *Annalen*. A Polish edition, by Prof. J. J. Boguski, was published at Warsaw in 1885 ; and permission has been asked and granted for the publication of an Italian edition.



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REDUCTION TO AND FROM C.G.S. MEASURES.

ACCORDING to Col. Clarke's comparisons of standards of length (printed in 1866), the metre is equal to 1·09362311 yard, or 3·2808693 feet, or 39·370432 inches, the standard metre being taken as correct at 0° C., and the standard yard as correct at 16 $\frac{2}{3}$ ° C. Hence the inch is 2·5399772 centims., the foot 30·479726 centims., the square inch 6·4514842 square centims., and the cubic inch 16·3866227 cubic centims. According to the U.S. Coast Survey Bulletin, No. 9, 1889, a more probable value of the metre is 39·36980 inches.

According to the comparison made by Professor W. H. Miller in 1844 of the "kilogramme des Archives," the standard of French weights, with two English pounds of platinum, and additional weights, also of platinum, the kilogramme is 15432·34874 grains, of which the new standard pound contains 7000. Hence the kilogramme would be 2·2046212 pounds, and the pound 453·59265 grammes.

Three standard pounds, one of platinum-iridium and the other two of gilded bronze, belonging to the *Standards Department*, were compared, in 1883, at the *Bureau International des Poids et Mesures*, with standards belonging

to the *Bureau*, and their values in grammes were found to be respectively

453·59135,

453·58924,

453·58738.

—*Travaux et Mémoires*, tome IV.

In the following tables, *cm.* denotes *centimetre* or *centimetres*, *gm.* denotes *gramme* or *grammes*.

The numbers headed “reciprocals” are the factors for reducing *from* C.G.S. measures.

Length.

		<i>cm.</i>	Reciprocals.
1 inch, - - - =		2·5400	·39370
1 foot, - - - =		30·4797	·032809
1 yard, - - - =		91·4392	·010936
1 mile, - - - =		160933·	$6·2138 \times 10^{-6}$
1 sea mile, - - - =		185230·	$5·398 \times 10^{-6}$

Area.

		<i>sq. cm.</i>	Reciprocals.
1 sq. inch, - - - =		6·4515	·1550
1 sq. foot, - - - =		929·01	·001076
1 sq. yard, - - - =		8361·13	·0001196
1 sq. mile, - - - =		$2·59 \times 10^{10}$	$3·861 \times 10^{-11}$

Volume.

		<i>cub. cm.</i>	Reciprocals.
1 cub. inch, - - - =		16·387	·06102
1 cub. foot, - - - =		28316·	$3·532 \times 10^{-5}$
1 cubic yard, - - - =		764535·	$1·308 \times 10^{-6}$
1 pint, - - - =		567·63	·001762
1 gallon, - - - =		4541·	·0002202

Mass.

		<i>gm.</i>	Reciprocals.
1 grain, - - - =		·0647990	15·432
1 ounce avoird., - - - =		28·3495	·035274
1 pound „, - - - =		453·59	·0022046
1 ton, - - - =		$1·01605 \times 10^6$	$9·84206 \times 10^{-7}$

Velocity.

		cm. per sec.	Reciprocals.
1 knot,	- - - =	51·453	·019435
1 foot per sec.,	- - - =	30·4797	·032809
1 mile per hour,	- - - =	44·704	·022369
1 kilometre per hour,	- - - =	27·777	·036

The knot, according to the best nautical authorities, is a velocity, not a length. "Knots" is equivalent to "sea miles per hour."

Density.

		gm. per cub. cm.	Reciprocals.
1 grain per cub. inch,	- - - =	·00395438	252·88
1 lb. per cub. foot,	- - - =	·016019	62·426

The value of g assumed in the following tables is 981 :—

Force.

		gm.	Dynes.
1 poundal,	- - - =		13825·
1 pound,	- - - =	453·59	$4·45 \times 10^5$
1 grain,	- - - =	·064799	63·6
1 kilogramme,	- - - =	10^3	$9·81 \times 10^5$

The ratio of the poundal to the dyne is independent of g .

Stress.

		gm. per sq. cm.	Dynes per sq. cm.
1 lb. per sq. foot,	- - - =	·48826	479·
1 lb. per sq. inch,	- - - =	70·31	69000·
1 ton per sq. inch,	- - - =	$1·575 \times 10^5$	$1·545 \times 10^8$
1 inch mercury at 0°,	- - - =	34·534	33880·
30 " " "	- - - =	1036·	$1·0163 \times 10^6$
1 centim. " "	- - - =	13·596	13338·
76 " " "	- - - =	1033·3	$1·0136 \times 10^6$

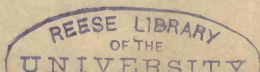
Surface Tension.

		gm. per cm.	Dynes per cm.
1 grain per linear inch,	- - - =	·02551	25·
1 lb. " " foot,	- - - =	14·88	14600·

Work and Energy.

		gm.-centims.	Ergs.
1 foot-pound,	- - - =	13825·	$1·3562 \times 10^7$
1 foot-poundal,	- - - =		$4·2139 \times 10^5$
1 foot-grain,	- - - =	1·975	$1·9375 \times 10^3$
1 foot-ton,	- - - =	$3·097 \times 10^7$	$3·0380 \times 10^{10}$
1 kilogrammetre,	- - - =	10^5	$9·81 \times 10^7$
1 joule,	- - - =		10^7

The value of the foot-poundal in ergs is independent of g .



Rate of Working.

	gm.-cm. per sec.	Ergs per sec.
1 horse-power, - - - =	$7\cdot604 \times 10^6$	$7\cdot46 \times 10^9$
1 force-de-cheval, - - - =	$7\cdot5 \times 10^6$	$7\cdot36 \times 10^9$
1 kilowatt, - - - =		10^{10}
1 watt, - - - =		10^7

Mechanical Equivalents of Heat.

(For water at 10° C. or 50° F.)

	gm. centims.	Ergs
1 gramme through 1° C.,	$4\cdot281 \times 10^4$	$4\cdot2 \times 10^7$
1 pound through 1° C.,	$1\cdot942 \times 10^7$	$1\cdot905 \times 10^{10}$
1 pound through 1° F.,	$1\cdot079 \times 10^7$	$1\cdot058 \times 10^{10}$

Various Measures of Length, in centimetres.

French foot, 32·484 (= 12 inches = 144 lines).

Toise, 194·904 (= 6 feet).

Rhenish or Prussian foot, 31·385; Austrian foot, 31·611; Bavarian, 29·186; Hanoverian, 29·209; Saxon, 28·319; Hessian, 28·770; Wurtemberg, 28·649; Baden, 30·000; Russian foot, 30·47945.

The Russian sagène or sashen is 7 feet.

Verst, 106678 (= 500 sashen).

Prussian mile, 753250 (= 24000 feet).

Austrian mile, 758666 (= 24000 feet).

Geographical mile as understood in Germany, 742040.

Various Measures of Mass, in grammes.

Zollverein pound, 500; Prussian pound, 467·711; Austrian, 560·012; Russian, 409·52.

Each of these pounds is divided into 32 loth or lot, and 100 pounds make one centner.



CHAPTER I.

GENERAL THEORY OF UNITS.

Units and Derived Units.

1. THE *numerical value* of a concrete quantity is its ratio to a selected magnitude of the same kind, called *the unit*.

Thus, if L denote a definite length, and l the unit length, $\frac{L}{l}$ is a ratio in the strict Euclidean sense, and is called the numerical value of L .

The numerical value of a concrete quantity varies directly as the concrete quantity itself, and inversely as the unit in terms of which it is expressed.

2. A unit of one kind of quantity is sometimes defined by reference to a unit of another kind of quantity. For example, the unit of *area* is commonly defined to be the area of the square described upon the unit of length; and the unit of *volume* is commonly defined as the volume of the cube constructed on the unit of length. The units of area and volume thus defined are called *derived units*, and are more convenient for calculation than independent units would be. For example, when the above

definition of the unit of area is employed, we can assert that [the numerical value of] the area of any rectangle is equal to the product of [the numerical values of] its length and breadth ; whereas, if any other unit of area were employed we should have to introduce a third factor which would be constant for all rectangles.

3. Still more frequently, a unit of one kind of quantity is defined by reference to two or more units of other kinds. For example, the unit of *velocity* is commonly defined to be that velocity with which the unit length would be described in the unit time. When we specify a velocity as so many *miles per hour*, or so many *feet per second*, we in effect employ as the unit of velocity a *mile per hour* in the former case, and a *foot per second* in the latter. These are *derived units* of velocity.

Again, the unit *acceleration* is commonly defined to be that acceleration with which a unit of velocity would be gained in a unit of time. The unit of acceleration is thus derived directly from the units of velocity and time, and therefore indirectly from the units of length and time.

4. In these and all other cases, the practical advantage of employing derived units is, that we thus avoid the introduction of additional factors, which would involve needless labour in calculating and difficulty in remembering.*

5. The correlative term to *derived* is *fundamental*.

* An example of such needless factors may be found in the rules commonly given in English books for finding the mass of a body when its volume and material are given. "Multiply the volume in cubic feet by the specific gravity and by 62·4, and the product will be the mass in pounds ;" or "multiply the volume in cubic

Thus, when the units of area, volume, velocity, and acceleration are defined as above, the units of length and time are called the fundamental units.

Dimensions.

6. Let us now examine the laws according to which derived units vary when the fundamental units are changed.

Let V denote a concrete velocity such that a concrete length L is described in a concrete time T ; and let v, l, t denote respectively the unit velocity, the unit length, and the unit time.

The numerical value of V is to be equal to the numerical value of L divided by the numerical value of T . But these numerical values are $\frac{V}{v}, \frac{L}{l}, \frac{T}{t}$;

hence we must have

$$\frac{V}{v} = \frac{L}{l} \frac{t}{T} \cdot \cdot \cdot \cdot \cdot \cdot \quad (1)$$

This equation shows that, when the units are changed (a change which does not affect V, L , and T), v must vary directly as l and inversely as t ; that is to say, *the unit of velocity varies directly as the unit of length, and inversely as the unit of time.*

Equation (1) also shows that *the numerical value $\frac{V}{v}$ of a given velocity varies inversely as the unit of length, and directly as the unit of time.*

inches by the specific gravity and by 253, and the product will be the mass in grains." The factors 62·4 and 253 here employed would be avoided—that is, would be replaced by unity, if the unit volume of water were made the unit of mass.

7. Again, let A denote a concrete acceleration such that the velocity V is gained in the time T' , and let a denote the unit of acceleration. Then, since the numerical value of the acceleration A is the numerical value of the velocity V divided by the numerical value of the time T' , we have

$$\frac{A}{a} = \frac{V}{v} \frac{t}{T'}.$$

But by equation (1) we may write $\frac{L}{l} \frac{t}{T}$ for $\frac{V}{v}$. We thus obtain

$$\frac{A}{a} = \frac{L}{l} \frac{t}{T} \frac{t}{T'}. \quad \dots \dots \dots (2)$$

This equation shows that when the units a , l , t are changed (a change which will not affect A , L , T or T'), a must vary directly as l , and inversely in the duplicate ratio of t ; and the numerical value $\frac{A}{a}$ will vary inversely as l , and directly in the duplicate ratio of t . In other words, *the unit of acceleration varies directly as the unit of length, and inversely as the square of the unit of time; and the numerical value of a given acceleration varies inversely as the unit of length, and directly as the square of the unit of time.*

It will be observed that these have been deduced as direct consequences from the fact that [the numerical value of] an acceleration is equal to [the numerical value of] a length, divided by [the numerical value of] a time, and then again by [the numerical value of] a time.

The relations here pointed out are usually expressed by

saying that *the dimensions of acceleration** are $\frac{\text{length}}{(\text{time})^2}$, or that *the dimensions of the unit of acceleration** are

$$\frac{\text{unit of length}}{(\text{unit of time})^2}$$

8. We have treated these two cases very fully, by way of laying a firm foundation for much that is to follow. We shall hereafter use an abridged form of reasoning, such as the following :—

$$\text{velocity} = \frac{\text{length}}{\text{time}};$$

$$\text{acceleration} = \frac{\text{velocity}}{\text{time}} = \frac{\text{length}}{(\text{time})^2}.$$

Such equations as these may be called *dimensional equations*. Their full interpretation is obvious from what precedes. In all such equations, constant numerical factors can be discarded, as not affecting dimensions.

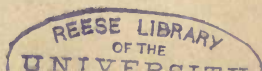
9. As an example of the application of equation (2) we shall compare the unit acceleration based on the foot and second with the unit acceleration based on the yard and minute.

Let l denote a foot, L a yard, t a second, T a minute, T' a minute. Then a will denote the unit acceleration based on the foot and second, and A will denote the unit

* Professor James Thomson ('Brit. Assoc. Report,' 1878, p. 452) objects to these expressions, and proposes to substitute the following :—

$$\text{“ Change-ratio of unit of acceleration} = \frac{\text{change-ratio of unit of length}}{(\text{change-ratio of unit of time})^2} \text{”}$$

This is very clear and satisfactory as a full statement of the meaning intended ; but it is necessary to tolerate some abridgment of it for practical working.



acceleration based on the yard and minute. Equation (2) becomes

$$\frac{A}{a} = \frac{3}{1} \times \left(\frac{1}{60}\right)^2 = \frac{1}{1200}; \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (3)$$

that is to say, an acceleration in which a yard per minute of velocity is gained per minute, is $\frac{1}{1200}$ of an acceleration in which a foot per second is gained per second.

Meaning of "per."

10. The word *per*, which we have several times employed in the present chapter, denotes division of the quantity named before it by the quantity named after it. Thus, to compute velocity in feet per second, we must divide a number of feet by a number of seconds.*

If velocity is continuously varying, let x be the number of feet described since a given epoch, and t the number of seconds elapsed, then $\frac{dx}{dt}$ is what is meant by the number of feet *per* second. The word should never be employed in the specification of quantities, except when the quantity named before it varies directly as the quantity named after it, at least for small variations—as, in the above instance, the distance described is ultimately proportional to the time of describing it.

*Extended Sense of the terms "Multiplication" and
"Division."*

11. In ordinary multiplication the multiplier is always

* It is not correct to speak of interest at the rate of *Five Pounds per cent.* It should be simply *Five per cent.* A rate of five pounds in every hundred pounds is not different from a rate of five shillings in every hundred shillings.

a mere numerical quantity, and the product is of the same nature as the multiplicand. Hence in ordinary division either the divisor is a mere numerical quantity and the quotient a quantity of the same nature as the dividend ; or else the divisor is of the same nature as the dividend, and the quotient a mere numerical quantity.

But in discussing problems relating to units, it is convenient to extend the meanings of the terms "multiplication" and "division." A distance divided by a time will denote a velocity—the velocity with which the given distance would be described in the given time. The distance can be expressed as a unit distance multiplied by a numerical quantity, and varies jointly as these two factors ; the time can be expressed as a unit time multiplied by a numerical quantity, and is jointly proportional to *these* two factors. Also, the velocity remains unchanged when the time and distance are both changed in the same ratio.

12. The three quotients

$$\frac{1 \text{ mile}}{1 \text{ hour}} \quad \frac{5280 \text{ ft.}}{3600 \text{ sec.}} \quad \frac{22 \text{ ft.}}{15 \text{ sec.}}$$

all denote the same velocity, and are therefore to be regarded as equal. In passing from the first to the second, we have changed the units in the inverse ratio to their numerical multipliers, and have thus left both the distance and the time unchanged. In passing from the second to the third, we have divided the two numerical factors by a common measure, and have thus changed the distance and the time in the same ratio. A change in either factor of the numerator will be compensated by a proportional change in either factor of the denominator.

Further, since the velocity $\frac{22 \text{ ft.}}{15 \text{ sec.}}$ is $\frac{22}{15}$ of the velocity $\frac{1 \text{ ft.}}{1 \text{ sec.}}$, we are entitled to write $\frac{22 \text{ ft.}}{15 \text{ sec.}} = \frac{22}{15} \cdot \frac{\text{ft.}}{\text{sec.}}$, thus separating the numerical part of the expression from the units part.

In like manner we may express the result of § 9 by writing

$$\frac{\text{yard}}{(\text{minute})^2} = \frac{1}{1200} \cdot \frac{\text{foot}}{(\text{second})^2}.$$

Such equations as these may be called "physical equations," inasmuch as they express the equality of physical quantities, whereas ordinary equations express the equality of mere numerical values. The use of physical equations in problems relating to units is to be strongly recommended, as affording a natural and easy clue to the necessary calculations, and especially as obviating the doubt by which the student is often embarrassed as to whether he ought to multiply or divide.

13. In the following examples, which illustrate the use of physical equations, we shall employ l to denote the unit length, m the unit mass, and t the unit time.

Ex. 1. If the yard be the unit of length, and the acceleration of gravity (in which a velocity of 32.2 ft. per sec. is gained per sec.) be represented by 2415, find the unit of time.

We have $l = \text{yard}$, and

$$32.2 \frac{\text{ft.}}{(\text{sec.})^2} = 2415 \frac{l}{t^2} = 2415 \frac{3 \text{ ft.}}{t^2},$$

$$\therefore t^2 = \frac{2415 \times 3}{32.2} \text{ sec.}^2 = 225 \text{ sec.}^2, \quad t = 15 \text{ sec.}$$

Ex. 2. If the unit time be the second, the unit density 162 lbs. per cub. ft., and the unit force* the weight of an ounce at a place where g (in foot-second units) is 32, what is the unit length?

We have $t = \text{sec.}, \quad \frac{m}{l^3} = 162 \frac{\text{lb.}}{(\text{ft.})^3},$

and $\frac{ml}{\text{sec.}^2} = 32 \cdot \frac{\text{oz. ft.}}{\text{sec.}^2}, \text{ or } ml = 32 \text{ oz. ft.} = 2 \text{ lb. ft.}$

Hence by division

$$l^4 = \frac{1}{81} (\text{ft.})^4, \quad l = \frac{1}{3} \text{ ft.} = 4 \text{ in.}$$

Ex. 3. If the area of a field of 10 acres be represented by 100, and the acceleration of gravity (taken as 32 foot-second units) be $58\frac{2}{3}$, find the unit of time.

We have $48400 (\text{yd.})^2 = 100 l^2$, whence $l = 22 \text{ yd.};$
 and $32 \frac{\text{ft.}}{(\text{sec.})^2} = 58\frac{2}{3} \frac{l}{t^2} = \frac{176}{3} \frac{66 \text{ ft.}}{t^2},$
 whence $t^2 = \frac{176 \times 22}{32} \text{ sec.}^2 = 121 \text{ sec.}^2, \quad t = 11 \text{ sec.}$

Ex. 4. If 8 ft. per sec. be the unit velocity, and the acceleration of gravity (32 foot-second units) the unit acceleration, find the units of length and time.

We have the two equations

$$\frac{l}{t} = 8 \frac{\text{ft.}}{\text{sec.}}, \quad \frac{l}{t^2} = 32 \frac{\text{ft.}}{\text{sec.}^2},$$

whence by division $t = \frac{1}{4} \text{ sec.}$, and substituting this value of t in the first equation, we have $4 l = 8 \text{ ft.}, \quad l = 2 \text{ ft.}$

Ex. 5. If the unit force be 100 lbs. weight, the unit length 2 ft., and the unit time $\frac{1}{4} \text{ sec.}$, find the unit mass, the acceleration of gravity being taken as 32 foot-second units.

* For the dimensions of density and force, see § 14.

We have $l = 2$ ft., $t = \frac{1}{4}$ sec.,

$$100 \text{ lb. } 32 \frac{\text{ft.}}{\text{sec.}^2} = \frac{ml}{t^2} = \frac{m \text{ 2 ft.}}{\frac{1}{16} \text{ sec.}^2},$$

that is $100 \times 32 \text{ lb.} = 32 m$, $m = 100 \text{ lb.}$

Ex. 6. The number of seconds in the unit of time is equal to the number of feet in the unit of length, the unit of force is 750 lbs. weight [g being 32], and a cubic foot of the standard substance [substance of unit density] contains 13500 oz. Find the unit of time.

Let $t = x$ sec., then $l = x$ ft.; also let $m = y$ lb. Then we have

$$\frac{ml}{t^2} = \frac{y \text{ lb. } x \text{ ft.}}{x^2 \text{ sec.}^2} = \frac{y \text{ lb. ft.}}{x \text{ sec.}^2} = 750 \times 32 \frac{\text{lb. ft.}}{\text{sec.}^2}$$

or $\frac{y}{x} = 750 \times 32.$

Also $\frac{m}{l^3} = \frac{y \text{ lb.}}{x^3 \text{ ft.}^3} = 13500 \frac{\text{oz.}}{\text{ft.}^3};$

whence $\frac{y}{x^3} = 13500 \times \frac{1}{16}.$

Hence by division

$$x^2 = \frac{750 \times 32 \times 16}{13500} = \frac{16^2}{3^2}, \quad x = \frac{16}{3}, \quad t = \frac{16}{3} \text{ sec.}$$

Ex. 7. When an inch is the unit of length and t the unit of time, the measure of a certain acceleration is a ; when 5 ft. and 1 min. are the units of length and time respectively, the measure of the same acceleration is $10 a$. Find t .

Equating the two expressions for the acceleration, we have

$$a \frac{\text{inch}}{t^2} = 10 a \frac{5 \text{ ft.}}{(\text{min.})^2},$$

$$\text{whence } t^2 = (\text{min.})^2 \frac{\text{inch}}{50 \text{ ft.}} = \frac{(\text{min.})^2}{600} = 6 (\text{sec.})^2,$$

$$t = \sqrt{6} \text{ sec.}$$

Ex. 8. The numerical value of a certain force is 56 when the pound is the unit of mass, the foot the unit of length, and the second the unit of time; what will be the numerical value of the same force when the hundred-weight is the unit of mass, the yard the unit of length, and the minute the unit of time?

Denoting the required value by x , we have

$$56 \frac{\text{lb. ft.}}{\text{sec.}^2} = x \frac{\text{cwt. yard}}{\text{min.}^2};$$

$$x = 56 \frac{\text{lb. ft.}}{\text{cwt. yd.}} \left(\frac{\text{min.}}{\text{sec.}} \right)^2$$

$$= 56 \times \frac{1}{112} \times \frac{1}{3} \times 60^2 = 600.$$

Dimensions of Mechanical and Geometrical Quantities.

14. In the following list of dimensions, we employ the letters L, M, T as abbreviations for the words *Length*, *Mass*, *Time*. The symbol of equality is used to denote sameness of dimensions.

$$\text{Area} = L^2, \quad \text{Volume} = L^3, \quad \text{Velocity} = \frac{L}{T},$$

$$\text{Acceleration} = \frac{L}{T^2}, \quad \text{Momentum} = \frac{ML}{T}.$$

Density = $\frac{M}{L^3}$, density being defined as mass per unit volume.

Force = $\frac{ML}{T^2}$, since a force is measured by the momentum which it generates per unit of time, and is therefore.

the quotient of momentum by time—or since a force is measured by the product of a mass by the acceleration generated in this mass.

Work = $\frac{ML^2}{T^2}$, being the product of force and distance.

Kinetic energy = $\frac{ML^2}{T^2}$, being half the product of mass by square of velocity. The constant factor $\frac{1}{2}$ can be omitted, as not affecting dimensions.

Moment of couple = $\frac{ML^2}{T^2}$, being the product of a force by a length.

The dimensions of *angle*,* when measured by $\frac{\text{arc}}{\text{radius}}$, are zero. The same angle will be denoted by the same number, whatever be the unit of length employed. In fact we have $\frac{\text{arc}}{\text{radius}} = \frac{L}{L} = L^0$.

The work done by a couple in turning a body through any angle, is the product of the couple by the angle. The identity of dimensions between *work* and *couple* is thus verified.

$$\text{Angular velocity} = \frac{1}{T}.$$

$$\text{Angular acceleration} = \frac{1}{T^2}.$$

$$\text{Moment of inertia} = ML^2.$$

$$\text{Angular momentum} = \text{moment of momentum} = \frac{ML^2}{T},$$

* The name *radian* has been given to the angle whose arc is equal to radius. “An angle whose value in circular measure is θ ” is “an angle of θ radians.”

being the product of moment of inertia by angular velocity, or the product of momentum by length.

Intensity of pressure, or intensity of stress generally, being force per unit of area, is of dimensions $\frac{\text{force}}{\text{area}}$; that is, $\frac{M}{L^2 T^2}$.

Intensity of force of attraction at a point, often called simply *force at a point*, being force per unit of attracted mass, is of dimensions $\frac{\text{force}}{\text{mass}}$ or $\frac{L}{T^2}$. It is numerically equal to the acceleration which it generates, and has accordingly the dimensions of acceleration.

The *absolute force of a centre* of attraction, better called the *strength of a centre*, may be defined as the intensity of force at unit distance. If the law of attraction be that of inverse squares, the strength will be the product of the intensity of force at any distance by the square of this distance, and its dimensions will be $\frac{L^3}{T^2}$.

Curvature (of a curve) = $\frac{1}{L}$, being the angle turned by the tangent per unit distance travelled along the curve.

Tortuosity = $\frac{1}{L}$, being the angle turned by the osculating plane per unit distance travelled along the curve.

The *solid angle* or *aperture* of a conical surface of any form is measured by the area cut off by the cone from a sphere whose centre is at the vertex of the cone, divided by the square of the radius of the sphere. Its dimensions are therefore zero; or a solid angle is a numerical quantity independent of the fundamental units.

The *specific curvature* of a surface at a given point (Gauss's measure of curvature) is the solid angle described by a line drawn from a fixed point parallel to the normal at a point which travels on the surface round the given point, and close to it, divided by the very small area thus enclosed. Its dimensions are therefore $\frac{1}{L^2}$.

The *mean curvature* of a surface at a given point, in the theory of Capillarity, is the arithmetical mean of the curvatures of any two normal sections normal to each other. Its dimensions are therefore $\frac{1}{L}$.

CHAPTER II.

CHOICE OF THREE FUNDAMENTAL UNITS.

15. NEARLY all the quantities with which physical science deals can be expressed in terms of three fundamental units ; and the quantities commonly selected to serve as the fundamental units are

a definite length,
a definite mass,
a definite interval of time.

This particular selection is a matter of convenience rather than of necessity ; for any three independent units are theoretically sufficient. For example, we might employ as the fundamental units

a definite mass,
a definite amount of energy,
a definite density.

16. The following are the most important considerations which ought to guide the selection of fundamental units :—

(1) They should be quantities admitting of very accurate comparison with other quantities of the same kind.

(2) Such comparison should be possible at all times. Hence the standards must be permanent—that is, not liable to alter their magnitude with lapse of time.

(3) Such comparisons should be possible at all places. Hence the standards must not be of such a nature as to change their magnitude when carried from place to place.

(4) The comparison should be easy and direct.

Besides these experimental requirements, it is also desirable that the fundamental units be so chosen that the definition of the various derived units shall be easy, and their dimensions simple.

17. There is probably no kind of magnitude which so completely fulfils the four conditions above stated as a standard of *mass*, consisting of a piece of gold, platinum, or some other substance not liable to be affected by atmospheric influences. The comparison of such a standard with other bodies of approximately equal mass is effected by weighing, which is, of all the operations of the laboratory, the most exact. Very accurate copies of the standard can thus be secured; and these can be carried from place to place with little risk of injury.

The third of the requirements above specified forbids the selection of a *force* as one of the fundamental units. Forces at the same place can be very accurately measured by comparison with weights; but as gravity varies from place to place, the force of gravity upon a piece of metal, or other standard weight, changes its magnitude in travelling from one place to another. A spring-balance, it is true, gives a direct measure of

force; but its indications are too rough for purposes of accuracy.

18. *Length* is an element which can be very accurately measured and copied. But every measuring instrument is liable to change its length with temperature. It is therefore necessary, in defining a length by reference to a concrete material standard, such as a bar of metal, to state the temperature at which the standard is correct. The temperature now usually selected for this purpose is that of a mixture of ice and water (0° C.), observation having shown that the temperature of such a mixture is constant.

The length of the standard should not exceed the length of a convenient measuring-instrument; for, in comparing the standard with a copy, the shifting of the measuring-instrument used in the comparison introduces additional risk of error.

In *end-standards*, the standard length is that of the bar as a whole, and the ends are touched by the instrument every time that a comparison is made. This process is liable to wear away the ends and make the standard false. In *line-standards*, the standard length is the distance between two scratches, and the comparison is made by optical means. The scratches are usually at the bottom of holes sunk half-way through the bar.

19. *Time* is also an element which can be measured with extreme precision. The direct instruments of measurement are clocks and chronometers; but these are checked by astronomical observations, and especially by transits of stars. The time between two successive transits of a star is (very approximately) the time of the

earth's rotation on its axis ; and it is upon the uniformity of this rotation that the preservation of our standards of time depends.

Necessity for a Common Scale.

20. The existence of quantitative correlations between the various forms of energy, imposes upon men of science the duty of bringing all kinds of physical quantity to one common scale of comparison. Several such measures (called *absolute* measures) have been published in recent years ; and a comparison of them brings very prominently into notice the great diversity at present existing in the selection of particular units of length, mass, and time.

Sometimes the units employed have been the foot, the grain, and the second ; sometimes the millimetre, milligramme, and second ; sometimes the centimetre, gramme, and second ; sometimes the centimetre, gramme, and minute ; sometimes the metre, tonne, and second ; sometimes the metre, gramme, and second ; while sometimes a mixture of units has been employed ; the area of a plate, for example, being expressed in square metres, and its thickness in millimetres.

A diversity of scales may be tolerable, though undesirable in the specification of such simple matters as length, area, volume, and mass when occurring singly ; for the reduction of these from one scale to another is generally understood. But when the quantities specified involve a reference to more than one of the fundamental units, and especially when their dimensions in terms of these units are not obvious, but require careful working out,

there is great increase of difficulty and of liability to mistake.

A general agreement as to the particular units of length, mass, and time which shall be employed—if not in all scientific work, at least in all work involving complicated reference to units—is urgently needed; and almost any one of the selections above instanced would be better than the present option.

21. We shall adopt the recommendation of the Units Committee of the British Association (see Appendix), that all specifications be referred to the *Centimetre*, the *Gramme*, and the *Second*. The system of units derived from these as the fundamental units is called the *C.G.S. system*; and the units of the system are called the *C.G.S. units*.

The reason for selecting the centimetre and gramme, rather than the metre and gramme, is that, since a gramme of water has a volume of approximately 1 cubic centimetre, the former selection makes the density of water unity; whereas the latter selection would make it a million, and the density of a substance would be a million times its specific gravity instead of being identical with its specific gravity as in the C.G.S. system.

Even those who may have a preference for some other units will nevertheless admit the advantage of having a variety of results, from various branches of physics, reduced from their original multiplicity and presented in one common scale.

22. The adoption of one common scale for all quantities involves the frequent use of very large and very

small numbers. Such numbers are most conveniently written by expressing them as the product of two factors, one of which is a power of 10; and it is usually advantageous to effect the resolution in such a way that the exponent of the power of 10 shall be the characteristic of the logarithm of the number. Thus 3240000000 will be written 3.24×10^9 , and .00000324 will be written 3.24×10^{-6} .

CHAPTER III.

MECHANICAL UNITS.

Value of g.

23. ACCELERATION is defined as the rate of increase of velocity per unit of time. The C.G.S. unit of acceleration is the acceleration of a body whose velocity increases in every second by the C.G.S. unit of velocity—namely, by a centimetre per second. The apparent acceleration of a body falling freely under the action of gravity in vacuo is denoted by g . The value of g in C.G.S. units at any part of the earth's surface is approximately given by the following formula.

$$g = 980.6056 - 2.5028 \cos 2\lambda - .000\,003h,$$

λ denoting the latitude, and h the height of the station (in centimetres) above sea-level.

The constants in this formula have been deduced from numerous pendulum experiments in different localities, the length l of the seconds' pendulum being connected with the value of g by the formula $g = \pi^2 l$.

Dividing the above equation by π^2 we have, for the length of the seconds' pendulum, in centimetres,

$$l = 99.3563 - .2536 \cos 2\lambda - .000\,0003h.$$

At sea-level these formulæ give the following values for the places specified :—

	Latitude.	Value of g .	Value of l .
Equator, - - -	0 0	978·10	99·103
Latitude 45°, - -	45 0	980·61	99·356
Munich, - - -	48 9	980·88	99·384
Paris, - - -	48 50	980·94	99·390
Greenwich, - - -	51 29	981·17	99·413
Göttingen, - - -	51 32	981·17	99·414
Berlin, - - -	52 30	981·25	99·422
Dublin, - - -	53 21	981·32	99·429
Manchester, - - -	53 29	981·34	99·430
Belfast, - - -	54 36	981·43	99·440
Edinburgh, - - -	55 57	981·54	99·451
Aberdeen, - - -	57 9	981·64	99·461
Pole, - - -	90 0	983·11	99·610

The difference between the greatest and least values (in the case of both g and l) is about $\frac{1}{198}$ of the mean value.

24. The Standards Department of the Board of Trade, being concerned only with relative determinations, has adopted the formula

$$g = g_0(1 - 0.00257 \cos 2\lambda) \left(1 - \frac{5}{4} \frac{h}{R}\right),$$

λ denoting the latitude, h the height above sea-level, R the earth's radius, g_0 the value of g in latitude 45° at sea-level, which may be treated as an unknown constant multiplier. Putting for R its value in centimetres, the formula gives

$$g = g_0(1 - 0.00257 \cos 2\lambda - 1.96h \times 10^{-9}),$$

where h denotes the height in centimetres.

The formula which we employed in the preceding section gives

$$g = g_0(1 - \cdot 00255 \cos 2\lambda)\left(1 - \frac{2h}{R}\right).$$

As regards the factor dependent on height, theory indicates $1 - \frac{2h}{R}$ as its correct value for such a case as that of a balloon in mid-air over a low-lying country; the value $1 - \frac{5}{4} \frac{h}{R}$ may be accepted as more correct for an elevated plateau on the earth's surface.

Force.

25. The C.G.S. unit of force is called the *dyne*. It is the force which, acting upon a gramme for a second, generates a velocity of a centimetre per second.

It may otherwise be defined as the force which, acting upon a gramme, produces the C.G.S. unit of acceleration, or as the force which, acting upon any mass for 1 second, produces the C.G.S. unit of momentum.

To show the equivalence of these three definitions, let m denote mass in grammes, v velocity in centimetres per second, t time in seconds, F force in dynes.

Then, by the second law of motion, we have

$$\text{acceleration} = \frac{\text{force}}{\text{mass}};$$

that is, if a denote acceleration in C.G.S. units, $a = \frac{F}{m}$;

hence, when a and m are each unity, F will be unity.

Again, by the nature of uniform acceleration, we have $v = at$, v denoting the velocity due to the acceleration a , continuing for time t .

Hence we have $F = ma = \frac{mv}{t}$. Therefore, if $mv = 1$ and $t = 1$, we have $F = 1$.

As a particular case, if $m = 1$, $v = 1$, $t = 1$, we have $F = 1$.

26. The force represented by the *weight of a gramme* varies from place to place. It is the force required to sustain a gramme in vacuo, and would be nil at the earth's centre, where gravity is nil. To compute its amount in dynes at any place where g is known, observe that a mass of 1 gramme falls in vacuo with acceleration g . The force producing this acceleration (namely, the weight of the gramme) must be equal to the product of the mass and acceleration, that is, to g .

The weight (when weight means force) of 1 gramme is therefore g dynes; and the weight of m grammes is mg dynes.

27. Force is said to be expressed in *gravitation-measure* when it is expressed as equal to the weight of a given mass. Such specification is inexact unless the value of g is also given. For purposes of accuracy it must always be remembered that the pound, the gramme, etc., are, strictly speaking, units of mass. Such an expression as "a force of 100 tons" must be understood as an abbreviation for "a force equal to the weight [at the locality in question] of 100 tons."

28. The name *poundal* has recently been given to the unit force based on the pound, foot, and second; that is, the force which, acting on a pound for a second, generates a velocity of a foot per second. It is $\frac{1}{g}$ of the

weight of a pound, g denoting the acceleration due to gravity expressed in foot-second units, which is about 32.2 in Great Britain.

To compare the poundal with the dyne, let x denote the number of dynes in a poundal; then we have

$$x \frac{\text{gm. cm.}}{\text{sec.}^2} = \frac{\text{lb. ft.}}{\text{sec.}^2},$$

$$x = \frac{\text{lb.}}{\text{gm.}} \cdot \frac{\text{ft.}}{\text{cm.}} = 453.59 \times 30.4797 = 13825.$$

Work and Energy.

29. The C.G.S. unit of work is called the *erg*. It is the amount of work done by a dyne working through a distance of a centimetre.

The C.G.S. unit of energy is also the *erg*, energy being measured by the amount of work which it represents.

30. To establish a rule for computing the *kinetic energy* (or *energy due to the motion*) of a given mass moving with a given velocity, it is sufficient to consider the case of a body falling in vacuo.

When a body of m grammes falls through a height of h centimetres, the working force is the weight of the body—that is, gm dynes, which, multiplied by the distance worked through, gives gmh ergs as the work done. But the velocity required is such that $v^2 = 2gh$. Hence we have $gmh = \frac{1}{2}mv^2$.

The kinetic energy of a mass of m grammes moving with a velocity of v centimetres per second is therefore $\frac{1}{2}mv^2$ ergs; that is to say, this is the amount of work which would be required to generate the motion of the body, or is the amount of work which the body

would do against opposing forces before it would come to rest.

31. Work, like force, is often expressed in *gravitation-measure*. Gravitation units of work, such as the foot-pound and kilogramme-metre, vary with locality, being proportional to the value of g .

One gramme-centimetre is equal to g ergs.

One kilogramme-metre is equal to 100,000 g ergs.

One foot-poundal is $453.59 \times (30.4797)^2 = 421390$ ergs.

One foot-pound is 13,825 gramme-centims., which, if g be taken as 981, is 1.356×10^7 ergs.

32. The C.G.S. unit rate of working is 1 erg per second. Watt's "horse-power" is defined as 550 foot-pounds per second. This is 7.46×10^9 ergs per second. The "force de cheval" is defined as 75 kilogrammetres per second. This is 7.36×10^9 ergs per second. We here assume $g = 981$.

A new unit of rate of working has been lately introduced for convenience in certain electrical calculations. It is called the *Watt*, and is defined as 10^7 ergs per second. A thousand watts make a *kilowatt*. The following tabular statement will be useful for reference.

1 Watt = 10^7 ergs per second = .00134 horse-power
= .737 foot-pounds per second = .1019 kilogrammetres per second.

1 Kilowatt = 1.34 horse-power.

1 Horse-power = 550 foot-pounds per second = 76.0 kilogrammetres per second = 746 watts = 1.01385 force de cheval.

1 Force de cheval = 75 kilogrammetres per second
= 542.48 foot-pounds per second = 736 watts
= .9863 horse-power.

In connection with the Watt, a new unit of work has been introduced, called the *Joule*. It is 10^7 ergs.

Examples.

1. If a spring balance is graduated so as to show the masses of bodies in pounds or grammes when used at the equator, what will be its error when used at the poles, neglecting effects of temperature ?

Ans. Its indications will be too high by about $\frac{1}{196}$ of the total weight.

2. A cannon-ball, of 10,000 grammes, is discharged with a velocity of 45,000 centims. per second. Find its kinetic energy.

Ans. $\frac{1}{2} \times 10000 \times (45000)^2 = 1.0125 \times 10^{13}$ ergs.

3. In last question find the mean force exerted upon the ball by the powder, the length of the barrel being 200 centims.

Ans. 5.0625×10^{10} dynes.

4. Given that 42 million ergs are equivalent to 1 gramme-degree of heat, and that a gramme of lead at 10° C. requires 15.6 gramme-degrees of heat to melt it ; find the velocity with which a leaden bullet must strike a target that it may just be melted by the collision, supposing all the mechanical energy of the motion to be converted into heat and to be taken up by the bullet.

We have $\frac{1}{2}v^2 = 15.6 \times J$, where $J = 42 \times 10^6$. Hence $v^2 = 1310$ millions ; $v = 36.2$ thousand centims. per second.

5. With what velocity must a stone be thrown vertically upwards at a place where g is 981 that it may rise to a height of 3000 centims. ? and to what height would

it ascend if projected vertically with this velocity at the surface of the moon, where g is 150 ?

Ans. 2426 centims. per second ; 19620 centims.

Centrifugal Force.

33. A body moving in a curve must be regarded as continually falling away from a tangent. The acceleration with which it falls away is $\frac{v^2}{r}$, v denoting its velocity and r the radius of curvature. The acceleration of a body in any direction is always due to force urging it in that direction, this force being equal to the product of mass and acceleration. Hence the normal force on a body of m grammes moving in a curve of radius r centimetres, with velocity v centimetres per second, is $\frac{mv^2}{r}$ dynes. This force is directed towards the centre of curvature. The equal and opposite force with which the body reacts is called centrifugal force.

If the body moves uniformly in a circle, the time of revolution being T seconds, we have $v = \frac{2\pi r}{T}$; hence $\frac{v^2}{r} = \left(\frac{2\pi}{T}\right)^2 r$, and the force acting on the body is $mr\left(\frac{2\pi}{T}\right)^2$ dynes.

If n revolutions are made per minute, the value of T is $\frac{60}{n}$, and the force is $mr\left(\frac{n\pi}{30}\right)^2$ dynes.

Examples.

1. A body of m grammes moves uniformly in a circle of radius 80 centims., the time of revolution being $\frac{1}{4}$ of a

second. Find the centrifugal force, and compare it with the weight of the body.

Ans. The centrifugal force is $m \times \left(\frac{2\pi}{\frac{1}{4}}\right)^2 \times 80 = m \times 64\pi^2 \times 80 = 50532 \text{ } m \text{ dynes.}$

The weight of the body (at a place where g is 981) is 981 m dynes. Hence the centrifugal force is about $52\frac{1}{2}$ times the weight of the body.

2. At a bend in a river, the velocity in a certain part of the surface is 170 centims. per second, and the radius of curvature of the lines of flow is 9100 centims. Find the slope of the surface in a section transverse to the lines of flow.

Ans. Here the centrifugal force for a gramme of the water is $\frac{(170)^2}{9100} = 3.176$ dynes. If g be 981 the slope will

be $\frac{3.176}{981} = \frac{1}{309}$; that is, the surface will slope upwards from the concave side at a gradient of 1 in 309. The general rule applicable to questions of this kind is that the resultant of centrifugal force and gravity must be normal to the surface.

3. An open vessel of liquid is made to rotate rapidly round a vertical axis. Find the number of revolutions that must be made per minute in order to obtain a slope of 30° at a part of the surface distant 10 centims. from the axis, the value of g being 981.

Ans. We must have $\tan 30^\circ = \frac{f}{g}$, where f denotes the intensity of centrifugal force—that is, the centrifugal force per unit mass. We have therefore

$$981 \tan 30^\circ = 10 \left(\frac{n\pi}{30} \right)^2, \quad n \text{ denoting the number of revolutions per minute,}$$

$$= \frac{n^2 \pi^2}{90}.$$

Hence $n = 91.9$.

4. For the intensity of centrifugal force at the equator due to the earth's rotation, we have r = earth's radius = 6.38×10^8 , $T = 86164$, being the number of seconds in a sidereal day.

$$\therefore f = r \left(\frac{2\pi}{T} \right)^2 = 3.39.$$

This is about $\frac{1}{289}$ of the value of g .

If the earth were at rest, the value of g at the equator would be greater than at present by this amount. If the earth were revolving about 17 times as fast as at present, the value of g at the equator would be nil.

SUPPLEMENTAL SECTION.

On the help to be derived from Dimensions in investigating Physical Formulæ.

When one physical quantity is known to vary as some power of another physical quantity, it is often possible to find the exponent of this power by reasoning based on dimensions, and thus to anticipate the results—or some of the results—of a dynamical investigation.

Examples.

1. The time of vibration of a simple pendulum in a small arc depends on the length of the pendulum and the intensity of gravity. If we assume it to vary as the m^{th}

power of the length, and as the n^{th} power of g , and to be independent of everything else, the dimensions of a time must equal the m^{th} power of a length, multiplied by the n^{th} power of an acceleration, that is

$$\begin{aligned} T &= L^m (LT^{-2})^n = L^m L^n T^{-2n} \\ &= L^{m+n} T^{-2n}. \end{aligned}$$

Since the dimensions of both members are to be identical, we have, by equating the exponents of T ,

$$1 = -2n, \text{ whence } n = -\frac{1}{2},$$

and by equating the exponents of L ,

$$m + n = 0, \text{ whence } m = \frac{1}{2};$$

that is, the time of vibration varies directly as the square root of the length, and inversely as the square root of g .

2. The velocity of sound in a gas depends only on the density D of the gas and its coefficient of elasticity E , and we shall assume it to vary as $D^m E^n$.

The dimensions of velocity are LT^{-1} .

The dimensions of density, or $\frac{\text{mass}}{\text{volume}}$, are ML^{-3} .

The dimensions of E , which will be explained in the chapter on stress and strain, are $\frac{\text{force}}{\text{area}}$, or $(MLT^{-2})L^{-2}$, or $ML^{-1}T^{-2}$.

The equation of dimensions is

$$\begin{aligned} LT^{-1} &= M^m L^{-3m} \cdot M^n L^{-n} T^{-2n}, \\ &= M^{m+n} L^{-3m-n} T^{-2n}, \end{aligned}$$

whence, by equating coefficients, we have the three equations

$$1 = -3m - n, \quad -1 = -2n, \quad m + n = 0,$$

to determine the two unknowns m and n .

The second equation gives at once

$$n = \frac{1}{2}.$$

The third then gives

$$m = -\frac{1}{2},$$

and these values will be found to satisfy the first equation also.

The velocity, then, varies directly as the square root of E , and inversely as the square root of D .

3. The frequency of vibration f for a musical string (that is, the number of vibrations per unit time) depends on its length l , its mass m , and the force with which it is stretched F .

The dimensions of f are T^{-1} .

„ „ „ F „ MLT^{-2} .

Assume that f varies as $l^x m^y F^z$. Then we have

$$\begin{aligned} T^{-1} &= L^x M^y M^z L^z T^{-2z}, \\ &= L^{x+z} M^{y+z} T^{-2z}, \end{aligned}$$

giving $-1 = -2z$, $x + z = 0$, $y + z = 0$;

whence $z = \frac{1}{2}$, $x = -\frac{1}{2}$, $y = -\frac{1}{2}$.

Hence f varies as $\sqrt{\frac{F}{lm}}$.

4. The angular acceleration of a uniform disc round its axis depends on the applied couple G , the mass of the disc M , and its radius R .

Assume it to vary as $G^x M^y R^z$.

The dimensions of angular acceleration are T^{-2} .

„ „ „ G „ ML^2T^{-2} .
 „ „ „ R „ L

Hence we have

$$\begin{aligned} T^{-2} &= M^x L^{2x} T^{-2x} M^y L^z, \\ &= M^{x+y} L^{2x+z} T^{-2x}; \end{aligned}$$

giving $-2 = -2x, \quad x + y = 0, \quad 2x + z = 0,$
 whence $x = 1, \quad y = -1, \quad z = -2.$

Hence the angular acceleration varies as $\frac{G}{MR^2}.$

In the following example the information obtained is less complete.

5. The range of a projectile on a horizontal plane through the point of projection depends on the initial velocity V , the intensity of gravity g , and the angle of elevation α .

The dimensions of range are L .

„ „ V „ $LT^{-1}.$

„ „ g „ $LT^{-2}.$

„ „ α „ L^0T^0 , and the dimensions of all powers of α are L^0T^0 . Hence we can draw no inferences as to the manner in which α enters the expression for the range. The dimensions of this expression will depend upon V and g alone.

Assume that the range varies as $V^m g^n$. Then

$$L = (LT^{-1})^m (LT^{-2})^n,$$

$$= L^{m+n} T^{-m-2n};$$

giving $m + n = 1, \quad m + 2n = 0,$

whence $m = 2, \quad n = -1.$

Hence the range varies as $\frac{V^2}{g}$ when α is given.

CHAPTER IV.

HYDROSTATICS.

34. IN the C.G.S. system, density is expressed in grammes per cubic centim.

Defining the specific gravity of a substance as the ratio of its density in its actual condition to the density of pure water at 4° C., specific gravity is so nearly identical with density in the C.G.S. system that it is uncertain which is the greater.

According to the observations of Kupffer, reduced by Prof. W. H. Miller, the density of pure water at 4° C. is 1.000013. According to the observations of Tralles, reduced by Broch, it is .99988. A fresh determination is in progress at the *Bureau International des Poids et Mesures*.

As regards the density of pure water at 62° F. ($16^{\circ}\frac{2}{3}$ C.) in British measures, there is still wider divergence of authorities.

Rankine, at p. 99 of Rules and Tables, says 277.123 cubic inches "is the correct volume of 10 lbs. of pure water at 62° Fahr., and is therefore the true value of a gallon in cubic inches. By a former Act of Parliament, since repealed, a gallon was declared to be 277.274 cubic inches."

To find the density in grains per cubic inch, we must divide these numbers into 70,000. We thus obtain 252·595 from the value adopted by Rankine, and 252·458 from the erroneous value in the repealed Act.

Mr. H. J. Chaney, Warden of the Standards, in the Proceedings of the Royal Society for 1890, No. 294, p. 230, says the hitherto accepted value is 252·458, and gives a new determination from his own measurements, which is 252·286.

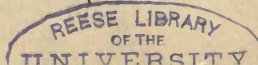
This value of Mr. Chaney's is equivalent to ·997643 gm. per cub. centim.; and as Mr. Chaney adopts ·998881 for the ratio of the volumes at 4° C. and 62° F., the density at 4° C. resulting from this determination is ·998752 gm. per cub. centim., which differs from the theoretical value unity by ·00125—a departure ten times as great as that found by Tralles and Broch, or 100 times as great as that found in the opposite direction by Kupffer and Miller.

35. The table on next page gives the volume of pure water at temperatures from 0° to 100° C. in terms of the volume at 0° C. To compare with the volume at 4° C. it is necessary to add ·00017. The values from 0° to 30° are taken from Broch's table, and those from 35° to 100° from a comparison of Rossetti's and Volkmann's.

Herr's formula for the volume at t° C., t being between 0 and 30, in terms of the volume at 0° C. is

$$1 - \cdot 000\ 0603\ t + \cdot 000\ 007\ 93\ t^2 - \cdot 000\ 000\ 0426\ t^3.$$

The ratio of the density at 4° C. to the density at 62° F. ($16^{\circ}\frac{2}{3}$ C.) is 1·001118 according to the above table, and this is also the value adopted by Rankine (Rules and



Tables, p. 146). Chaney adopts the reciprocal of ·998881, which is 1·001120.

Temp. Cent.	Volume.	Temp. Cent.	Volume.	Temp. Cent.	Volume.
0	1·000000	15	1·000735	30	1·00418
1	·999948	16	890	35	1·0057
2	911	17	1·001057	40	76
3	889	18	235	45	97
4	883	19	424	50	1·0118
5	891	20	624	55	142
6	914	21	835	60	168
7	952	22	1·002057	65	195
8	1·000003	23	289	70	1·0225
9	068	24	530	75	256
10	147	25	78	80	288
11	239	26	1·00304	85	1·0321
12	344	27	31	90	356
13	462	28	59	95	392
14	593	29	88	100	1·0430

36.

TABLE OF DENSITIES.

Solids.

(For the most part rough averages.)

Aluminium,	2·6	Carbon (diamond), ...	3·5
Antimony,	6·7	„ (graphite), ...	2·3
Bismuth,	9·8	„ (gas carbon),	1·9
Brass,	8·4	„ (wood charcoal),	1·6
Copper,	8·9	Phosphorus (ordi-	
Gold,	19·3	nary),	1·83
Iron,	7·8	„ (red), ...	2·2
Lead,	11·3	Sulphur (roll),	2·0
Nickel,	8·9	Quartz (rock cry-	
Platinum,	21·5	stal),	2·6512
Silver,	10·5	Sand (dry), ...	1·42
Sodium,	·98	Clay,	1·9
Tin,	7·3	Brick, ..	2·1
Zinc,	7·1	Basalt,	3·0
Cork, ..	·24	Chalk,	1·8 to 2·8
Oak, ...	·7 to 1·0	Glass (crown),	2·5 to 2·7
Ebony,	1·1 to 1·2	„ (flint),	3·0 to 3·5
Ice,	·917	Porcelain,	2·4

Liquids at 0° C.

Sea water,.....	1·026	Sulphuric Acid,	1·85
Alcohol,.....	·806	Nitric Acid,	1·56
Chloroform,	1·527	Hydrochloric Acid,..	1·27
Ether,.....	·736	Milk,	1·03
Bisulphide of Carbon,	1·293	Oil of Turpentine, ...	·87
Glycerine,	1·27	„ Linseed,	·94
Mercury,.....	13·596	„ Mineral,	·76 to ·83

More exactly, the density of mercury at 0° C., as compared with water at the temperature of maximum density, under atmospheric pressure, is 13·5956.

37. If a body weighs m grammes in vacuo and m' grammes in water of density unity, the volume of the body is $m - m'$ cubic centims. ; for the mass of the water displaced is $m - m'$ grammes, and each gramme of this water occupies a cubic centimetre.

Examples.

1. A glass cylinder, l centims. long, weighs m grammes in vacuo and m' grammes in water of unit density. Find its radius.

Solution. Its section is πr^2 , and is also $\frac{m - m'}{l}$; hence

$$r^2 = \frac{m - m'}{\pi l}.$$

2. Find the capacity at 0° C. of a bulb which holds m grammes of mercury at that temperature.

Solution. The specific gravity of mercury at 0° being 13·596 as compared with water at the temperature of maximum density, it follows that the mass of 1 cubic centim. of mercury is 13·596. Hence the required capacity is $\frac{m}{13·596}$ cubic centims.

3. Find the total pressure on a surface whose area is A square centims. when its centre of gravity is immersed to a depth of h centims. in water of unity density, atmospheric pressure being neglected.

Ans. Ah grammes weight; that is, gAh dynes.

4. If mercury of specific gravity 13·596 is substituted for water in the preceding question, find the pressure.

Ans. 13·596 Ah grammes weight; that is, 13·596 gAh dynes.

5. If h be 76, and A be unity in example 4, the answer becomes 1033·3 grammes weight, or 1033·3 g dynes.

For Paris, where g is 980·94, this is $1·0136 \times 10^6$ dynes.

Barometric Pressure.

38. The C.G.S. unit of pressure-intensity (that is, of pressure per unit area) is the pressure of a dyne per square centim.

At the depth of h centims. in a uniform liquid whose density is d [grammes per cubic centim.], the pressure due to the weight of the liquid is ghd dynes per square centim.

The pressure-intensity due to the weight of a column of mercury at 0° C., 76 centims. high, is found by putting $h=76$, $d=13·596$, and is 1033·3 g . It is therefore different at different localities. At Paris, where g is 980·94, it is $1·0136 \times 10^6$; that is, rather more than a megadyne* per square centim. To exert a pressure of exactly one megadyne per square centim., the height of the column at Paris must be 74·98 centims.

*The prefix *mega* denotes multiplication by a million. A megadyne is a force of a million dynes.

At Greenwich, where g is 981.17, the pressure due to 76 centims. of mercury at 0° C. is 1.0138×10^6 ; and the height which would give a pressure of 10^6 is 74.964 centims., or 29.513 inches.

Convenience of calculation would be promoted by adopting the pressure of a megadyne per square centim., or 10^6 C.G.S. units of pressure-intensity, as the standard atmosphere.

The standard now commonly adopted (whether 76 centims. or 30 inches) denotes different pressures at different places, the pressure denoted by it being proportional to the value of g .

We shall adopt the megadyne per square centim. as our standard atmosphere in the present work.

Examples.

1. What must be the height of a column of water of unit density to exert a pressure of a megadyne per square centim. at a place where g is 981?

$$\text{Ans. } \frac{1000000}{981} = 1019.4 \text{ centims.} \quad \text{This is } 33.445 \text{ feet.}$$

2. What is the pressure due to an inch of mercury at 0° C. at a place where g is 981? (An inch is 2.54 centims.)

$$\text{Ans. } 981 \times 2.54 \times 13.596 = 33878 \text{ dynes per square centim.}$$

3. What is the pressure due to a centim. of mercury at 0° C. at the same locality?

$$\text{Ans. } 981 \times 13.596 = 13338.$$

4. What is the pressure due to a kilometre of seawater of density 1.027, g being 981?

Ans. $981 \times 10^5 \times 1.027 = 1.0075 \times 10^8$ dynes per square centim., or 1.0075×10^2 megadynes per square centim. ; that is, about 100 atmospheres.

5. What is the pressure due to a mile of the same water ?

Ans. 1.6214×10^8 C.G.S. units, or 162.14 atmospheres [of a megadyne per square centim.].

Density of Air.

39. Regnault found that at Paris, under the pressure of a column of mercury at 0° , of the height of 76 centims., the density of perfectly dry air was .0012932 gramme per cubic centim. The pressure corresponding to this height of the barometer at Paris is 1.0136×10^6 dynes per square centim. Hence by Boyle's law, we can compute the density of dry air at 0° C. at any given pressure.

At a pressure of a megadyne (10^6 dynes) per square centim. the density will be $\frac{.0012932}{1.0136} = .0012759$.

The density of dry air at 0° C. at any pressure p (dynes per square centim.) is

$$p \times 1.2759 \times 10^{-9} (4)$$

Example.

Find the density of dry air at 0° C., at Edinburgh, under the pressure of a column of mercury at 0° C., of the height of 76 centims.

Here we have $p = 981.54 \times 76 \times 13.596 = 1.0142 \times 10^6$.

Ans. Required density = $1.2940 \times 10^{-3} = .0012940$ gramme per cubic centim.

40. *Absolute Densities of Gases, in grammes per cubic centim., at 0° C., and a pressure of 10⁶ dynes per square centim.*

	Mass of a cubic centim. in grammes.	Volume of a gramme in cubic centims.
Air, dry,	·0012759	783·8 -
Oxygen,	·0014107	708·9
Nitrogen,	·0012393	806·9
Hydrogen,	·00008837	11316·0 -
Carbonic Acid,	·0019509	512·6 -
,, Oxide,	·0012179	821·1
Marsh Gas,	·0007173	1394·1 -
Chlorine,	·0030909	323·5 -
Protoxide of Nitrogen,	·0019433	514·6
Binoxide ,,	·0013254	754·5
Sulphurous Acid,	·0026990	370·5
Cyanogen,	·0022990	435·0
Olefiant Gas,	·0012529	798·1
Ammonia,	·0007594	1316·8

The numbers in the second column are the reciprocals of those in the first.

The numbers in the first column are identical with the specific gravities referred to water as unity.

Assuming that the densities of gases at given pressure and temperature are directly as their atomic weights, we have for any gas at zero

$$pv\mu = 1 \cdot 1316 \times 10^{10} m;$$

v denoting its volume in cubic centims., m its mass in grammes, p its pressure in dynes per square centim., and μ its atomic weight referred to that of hydrogen as unity.

41. DEPARTURE FROM BOYLE'S LAW.

Regnault found that for most gases the product of the volume and pressure decreased as the pressure was increased from about 1 atm. to about 30 atm. When the

initial pressure was about 70 and the final pressure about 140 cm. of mercury, the ratio of the initial to the final value of VP had the following values, which are arranged in order of magnitude—

Air,.....	1·00215	Hydrochloric acid,...	1·00925
Binoxide of nitrogen,	1·00285	Sulphuretted hydrogen,	1·01083
Carbonic oxide,.....	1·00293	Ammonia,	1·01881
Marsh gas,.....	1·00634	Sulphurous acid,.....	1·02088
Protoxide of nitrogen,	1·00651	Cyanogen,	1·02353
Carbonic acid,.....	1·00722		

For hydrogen, the deviation from Boyle's law was in the opposite direction and smaller in amount. He summed up his results in the empirical formula

$$\frac{V_0 P_0}{VP} = 1 + A(m-1) + B(m-1)^2, \quad m \text{ denoting } \frac{V_0}{V}.$$

For air and for nitrogen A was negative and B positive. For carbonic acid both A and B were negative. For hydrogen both A and B were positive. The logarithms of their arithmetical values were

	Log A.	Log B.
Air,.....	$\bar{3}\cdot0435120$	$\bar{5}\cdot2873751$
Nitrogen,.....	$\bar{4}\cdot8389375$	$\bar{6}\cdot8476020$
Carbonic acid, ...	$\bar{3}\cdot9310399$	$\bar{6}\cdot8624721$
Hydrogen,.....	$\bar{4}\cdot7381736$	$\bar{6}\cdot9250787$

42. The researches of Natterer and the more recent researches of Cailletet and of Amagat have shown that when much higher pressures are employed the value of VP (for gases other than hydrogen) continues to decrease up to a certain point, which is different for different gases, and then continually increases more and more rapidly. Amagat places the minimum value of VP at 50 m. of mercury for nitrogen, 100 m. for oxygen, 65 m. for air, and 50 m. for carbonic oxide. The

following is a sample of Cailletet's results for nitrogen at 15°. They fix the minimum at about 60 m.

P in metres of mercury.	VP.	P.	VP.
39·359	8184	89·231	8323
49·271	8022	124·122	8857
59·462	7900	174·100	9191
64·366	7951	181·985	9330

For references, see *Jamin et Bouty*, tom. I., pp. 213-217.

Height of Homogeneous Atmosphere.

43. We have seen that the intensity of pressure at depth h , in a fluid of uniform density d , is ghd when the pressure at the upper surface of the fluid is zero.

The atmosphere is not a fluid of uniform density ; but it is often convenient to have a name to denote a height H such that $p = gHD$, where p denotes the pressure and D the density of the air at a given point.

It may be defined as the height of a column of uniform fluid having the same density as the air at the point, which would exert a pressure equal to that existing at the point.

If the pressure be equal to that exerted by a column of mercury of density 13·596 and height h , we have

$$p = gh \times 13\cdot596 ;$$

$$\therefore HD = h \times 13\cdot596, H = \frac{h \times 13\cdot596}{D}.$$

If it were possible for the whole body of air above the point to be reduced by vertical compression to the density which the air has at the point, the height from the point up to the summit of this compressed atmosphere would be equal to H , subject to a small correction for the variation of gravity with height.

H is called the *height of the homogeneous atmosphere* at the point considered. *Pressure-height* would be a better name.

The general formula for it is

$$H = \frac{p}{gD}; \quad . \quad . \quad . \quad (5)$$

and this formula will be applicable to any other gas as well as dry air, if we make D denote the density of the gas (in grammes per cubic centim.) at pressure p .

If, instead of p being given directly in dynes per square centim., we have given the height h of a column of liquid of density d which would exert an equal pressure, the formula reduces to

$$H = \frac{hd}{D}. \quad . \quad . \quad . \quad (6)$$

44. The value of $\frac{p}{D}$ in formula (5) depends only on the nature of the gas and on the temperature; hence, for a given gas at a given temperature, H varies inversely as g only.

For dry air at zero we have, by formula (4), p. 40,

$$\frac{p}{D} = \frac{10^9}{1.2759} = 7.8376 \times 10^8;$$

$$\therefore H = \frac{7.8376 \times 10^8}{g}.$$

At Paris, where g is 980.94, we find

$$H = 7.990 \times 10^5.$$

At Greenwich, where g is 981.17,

$$H = 7.988 \times 10^5.$$

Examples.

1. Find the height of the homogeneous atmosphere at Paris for dry air at 10°C ., and also at 100°C .

Ans. For given density, p varies as $1 + .00366 t$, t denoting the temperature on the Centigrade scale. Hence we have, at 10°C. ,

$$H = 1.0366 \times 7.99 \times 10^5 = 8.2824 \times 10^5;$$

and at 100°C. ,

$$H = 1.366 \times 7.99 \times 10^5 = 1.0914 \times 10^6.$$

2. Find the height of the homogeneous atmosphere for hydrogen at 0° , at a place where g is 981.

Here we have

$$H = \frac{p}{gd} = \frac{10^6}{981 \times 8.837 \times 10^{-5}} = 1.1535 \times 10^7.$$

Diminution of Density with increase of Height in the Atmosphere.

45. Neglecting the variation of gravity with height, the variation of H as we ascend in the atmosphere would depend only on variation of temperature. In an atmosphere of uniform temperature H will be the same at all heights. In such an atmosphere, an ascent of 1 centim. will involve a diminution of the pressure (and therefore of the density) by $\frac{1}{H}$ of itself, since the layer of air which

has been traversed is $\frac{1}{H}$ of the whole mass of superincumbent air. The density therefore diminishes by the same fraction of itself for every centim. that we ascend; in other words, the density and pressure diminish in geometrical progression as the height increases in arithmetical progression.

Denote height above a fixed level by x , and pressure by p . Then, in the notation of the differential calculus,

we have
$$\frac{dx}{H} = -\frac{dp}{p},$$

and if p_1 , p_2 are the pressures at the height x_1 , x_2 , we deduce

$$x_2 - x_1 = H \log_e \frac{p_1}{p_2} = H \times 2.3026 \log_{10} \frac{p_1}{p_2}. \quad (7)$$

In the barometric determination of heights it is usual to compute H by assuming a temperature which is the arithmetical mean of the temperatures at the two heights.

For the latitude of Greenwich formula (7) becomes

$$\begin{aligned} x_2 - x_1 &= (1 + .00366 t) 7.988 \times 10^5 \times 2.3026 \log \frac{p_1}{p_2} \\ &= (1 + .00366 t) 1,839,300 \log \frac{p_1}{p_2}, \quad (8) \end{aligned}$$

t denoting the mean temperature, and the logarithms being common logarithms.

To find the height at which the density would be halved, variations of temperature being neglected, we must put 2 for $\frac{p_1}{p_2}$ in these formulæ. The required height will be $H \log_e 2$, or, in the latitude of Greenwich, for temperature 0°C. , will be

$$1.8393 \times 10^6 \times .30103 = 553700.$$

The value of $\log_e 2$, or $2.3026 \log_{10} 2$, is

$$2.3026 \times .30103 = .69315.$$

Hence, by (7), for an atmosphere of any gas at uniform temperature, the height at which the density would be halved is the height of the homogeneous atmosphere for that gas, multiplied by .69315. The gas is assumed to obey Boyle's law.

Examples.

1. Show that, if the pressure of the gas at the lower station and the value of g be given, the height at which the density will be halved varies inversely as the density.

2. At what height, in an atmosphere of hydrogen at 0°C. , would the density be halved, g being 981?

Ans. 7.9955×10^6 .

Capillarity.

46. The phenomena of capillarity, soap-bubbles, etc., can be reduced to quantitative expression by assuming a tendency in the surface of every liquid to contract. The following table exhibits the intensity of this contractile force for various liquids at the temperature of 20°C. The contractile force diminishes as the temperature increases.

Superficial tensions at 20°C. , in dynes per linear centim., deduced from Quincke's results.

	Density.	Tension of Surface separating the Liquid from		
		Air.	Water.	Mercury.
Water, - - -	0.9982	81	0	418
Mercury, - - -	13.5432	540	418	0
Bisulphide of Carbon,	1.2687	32.1	41.75	372.5
Chloroform, - - -	1.4878	30.6	29.5	399
Alcohol, - - -	.7906	25.5	...	399
Olive Oil, - - -	.9136	36.9	20.56	335
Turpentine, - - -	.8867	29.7	11.55	250.5
Petroleum, - - -	.7977	31.7	27.8	284
Hydrochloric Acid, -	1.1	70.1	...	377
Solution of Hyposul- phite of Soda, - - }	1.1248	77.5	...	442.5

The values here given for water and mercury are only applicable when special precautions are taken to ensure cleanliness and purity. Without such precautions smaller values will be obtained. (Quincke in Wied. Ann., 1886, page 219.)

The following values are from the observations of A. M. Worthington (Proc. Roy. Soc., June 16, 1881), at temperatures from 15° to 18° C., for surfaces exposed to air :—

Surface Tension.

	In gm. per cm.	In dynes per cm.
Water,	·072 to ·080	70·6 to 78·5
Alcohol,	·02586	25·3
Turpentine,	·02818	27·6
Olive Oil,	·03373	33·1
Chloroform,	·03025	29·6

47. Very elaborate measurements of the thicknesses of soap films have been made by Reinold and Rücker (Phil. Trans., 1881, p. 456 ; and 1883, p. 651). When so thin as to appear black, the thickness varied from 7·2 to 14·5 millionths of a millimetre, the mean being 11·7. This is $1\cdot17 \times 10^{-6}$ centimetre. The following thicknesses were observed for the colours of the successive orders :—

	Thickness. cm.		Thickness. cm.
FIRST ORDER—		Yellow-Green, ..	$9\cdot64 \times 10^{-5}$
Red,	$2\cdot84 \times 10^{-5}$	Red,	10·52 ,,
SECOND ORDER—		FIFTH ORDER—	
Violet,	3·05 ,,	Green,	$1\cdot119 \times 10^{-4}$
Blue,	3·53 ,,	„	1·188 ,,
Green,	4·09 ,,	Red,	1·260 ,,
Yellow,	4·54 ,,	„	1·335 ,,
Orange,	4·91 ,,	SIXTH ORDER—	
Red,	5·22 ,,	Green,	1·410 ,,
THIRD ORDER—		„	1·479 ,,
Purple,	5·59 ,,	Red,	1·548 ,,
Blue,	5·77 ,,	„	1·627 ,,
„	6·03 ,,	SEVENTH ORDER—	
Green,	6·56 ,,	Green,	1·705 ,,
Yellow,	7·10 ,,	„	1·787 ,,
Red,	7·65 ,,	Red,	1·869 ,,
Bluish Red,	8·15 ,,	„	1·936 ,,
FOURTH ORDER—		EIGHTH ORDER—	
Green,	8·41 ,,	Green,	2·004 ,,
„	8·93 ,,	Red,	2·115 ,,

48. Depression of the barometrical column due to capillarity, according to Pouillet :—

Internal Diameter of tube.	Depression.	Internal Diameter of tube.	Depression.	Internal Diameter of tube.	Depression.
mm.	mm.	mm.	mm.	mm.	mm.
2	4·579	8·5	·604	15	·127
2·5	3·595	9	·534	15·5	·112
3	2·902	9·5	·473	16	·099
3·5	2·415	10	·419	16·5	·087
4	2·053	10·5	·372	17	·077
4·5	1·752	11	·330	17·5	·068
5	1·507	11·5	·293	18	·060
5·5	1·306	12	·260	18·5	·053
6	1·136	12·5	·230	19	·047
6·5	·995	13	·204	19·5	·041
7	·877	13·5	·181	20	·036
7·5	·775	14	·161	20·5	·032
8	·684	14·5	·143	21	·028

CHAPTER V.

STRESS, STRAIN, ELASTICITY, AND VISCOSITY.

49. IN the nomenclature introduced by Rankine, and adopted by Thomson and Tait, any change in the shape or size of a body is called a *strain*, and an action of force tending to produce a strain is called a *stress*. We shall always suppose strains to be small ; that is, we shall suppose the ratio of the initial to the final length of every line in the strained body to be nearly a ratio of equality.

50. A strain changes every small spherical portion of the body into an ellipsoid ; and the strain is said to be homogeneous when equal spherical portions in all parts of the body are changed into equal ellipsoids with their corresponding axes equal and parallel. When the strain consists in change of volume, unaccompanied by change of shape, the ellipsoids are spheres.

When strain is not homogeneous, but varies continuously from point to point, the *strain at any point* is defined by attending to the change which takes place in a very small sphere or cube having the point at its centre, so small that the strain throughout it may be regarded as homogeneous. In what follows we shall suppose strain to be homogeneous, unless the contrary is expressed.

51. The *axes of a strain* are the three directions in the body, at right angles to each other, which coincide with the directions of the axes of the ellipsoids. Lines drawn in the body in these three directions will remain at right angles to each other when the body is restored to its unstrained condition.

A cube with its edges parallel to the axes will be altered by the strain into a *rectangular* parallelepiped. Any other cube will be changed into an oblique parallelepiped.

When the axes have the same directions in space after as before the strain, the strain is said to be *unaccompanied by rotation*. When such parallelism does not exist, the strain is accompanied by rotation, namely, by the rotation which is necessary for bringing the axes from their initial to their final position.

The numbers which specify a strain are mere ratios, and are therefore independent of units.

52. When a body is under the action of forces which strain it, or tend to strain it; if we consider any plane section of the body, the portions of the body which it separates are pushing each other, pulling each other, or exerting some kind of force upon each other, across the section, and the mutual forces so exerted are equal and opposite. The specification of a stress must include a specification of these forces for all sections, and a body is said to be homogeneously stressed when these forces are the same in direction and intensity for all parallel sections. We shall suppose stress to be homogeneous, in what follows, unless the contrary is expressed.

53. When the force-action across a section consists of

a simple pull or push normal to the section, the direction of this simple pull or push (in other words, the normal to the section) is called an *axis of the stress*. A stress (like a strain) has always three axes, which are at right angles to one another. The mutual forces across a section not perpendicular to one of the three axes are in general partly normal and partly tangential—one side of the section is tending to slide past the other.

The force per unit area which acts across any section is called the intensity of the stress on this section, or simply the *stress on this section*. The dimensions of “force per unit area,” or $\frac{\text{force}}{\text{area}}$ are $\frac{M}{LT^2}$, which we shall therefore call the *dimensions of stress*.

54. The relation between the stress acting upon a body and the strain produced depends upon the elasticity* of the body, which requires in general 21 numbers for its complete specification. When the body has exactly the same properties in all directions, 2 numbers are sufficient. These specifying numbers are called *coefficients of elasticity*. A coefficient of elasticity expresses the quotient of a stress (of a given kind) by the strain (of a given kind) which it produces. Steel is an example of a body with large, and cork of a body with small, coefficients of elasticity.

In all cases (for solid bodies) equal and opposite strains (supposed small) require for their production equal and opposite stresses.

* The word *resilience*, which was employed in previous editions of this work, in place of *elasticity*, has strong recommendations; but engineers had already appropriated it in a different sense, and hence the proposed change has not found favour.

55. The coefficients of elasticity most frequently referred to are the three following :—

(1) Elasticity of volume, or resistance to hydrostatic compression. If V be the original and $V - v$ the strained volume, $\frac{v}{V}$ is called the compression, and when the body is subjected to uniform normal pressure P per unit area over its whole surface, the quotient of P by the compression is the elasticity of volume. This is the only kind of elasticity possessed by liquids and gases.

(2) Young's modulus, or the longitudinal elasticity of a body which is perfectly free to expand or contract laterally. In general, longitudinal extension produces lateral contraction, and longitudinal compression produces lateral extension. Let the unstrained length be L and the strained length $L \pm l$, then $\frac{l}{L}$ is taken as the measure of the longitudinal extension or compression. The stress on a cross section (that is, on a section to which the stress is normal) is called the longitudinal stress, and Young's modulus is the quotient of the longitudinal stress by the longitudinal extension or compression. If a wire of cross section A sq. cm. is stretched with a force of F dynes, and its length is thus altered from L to $L + l$, the value of Young's modulus for the wire is $\frac{F}{A} \cdot \frac{L}{l}$.

(3) "Simple rigidity" or resistance to shearing. This requires a more detailed explanation.

56. A shear may be defined as a strain by which a sphere of radius unity is converted into an ellipsoid of semiaxes $1, 1 + e, 1 - e$; in other words, it consists of an

extension in one direction combined with an equal compression in a perpendicular direction.

57. A unit square (Fig. 1) whose diagonals coincide with these directions is altered by the strain into a rhombus whose diagonals are $(1+e)\sqrt{2}$ and $(1-e)\sqrt{2}$, and whose area, being half the product of the diagonals, is $1-e^2$, or, to the first order of small quantities, is 1, the same as the area of the original square. The length of a side of the rhombus, being the square root of the

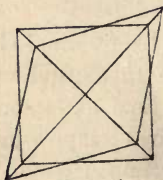


Fig. 1.



Fig. 2

sum of the squares of the semi-diagonals, is found to be $\sqrt{1+e^2}$ or $1+\frac{1}{2}e^2$, and is therefore, to the first order of small quantities, equal to a side of the original square.

58. To find the magnitude of the small angle which a side of the rhombus makes with the corresponding side of the square, we may proceed as follows:—Let acb (Fig. 2) be an enlarged representation of one of the small triangles in Fig. 1. Then we have $ab = \frac{1}{2}$, $cb = \frac{1}{2}e\sqrt{2} = \frac{e}{\sqrt{2}}$, angle $cba = \frac{\pi}{4}$. Hence the length of the perpendicular cd is $cb \sin \frac{\pi}{4} = \frac{e}{\sqrt{2}} \frac{1}{\sqrt{2}} = \frac{e}{2}$; and since ad is ultimately equal to ab , we have, to the first order of small quantities,

$$\text{angle } cab = \frac{cd}{ad} = \frac{\frac{1}{2}e}{\frac{1}{2}} = e.$$

The semi-angles of the rhombus are therefore $\frac{\pi}{4} \pm e$, and the angles of the rhombus are $\frac{\pi}{2} \pm 2e$; in other words, each angle of the square has been altered by the amount $2e$. *This quantity $2e$ is adopted as the measure of the shear.*

59. To find the perpendicular distance between opposite sides of the rhombus, we have to multiply a side by the cosine of $2e$, which, to the first order of small quantities, is 1. Hence the perpendicular distance between opposite sides of the square is not altered by the shear, and the relative movement of these sides is represented

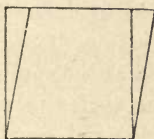


Fig 3.

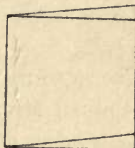


Fig. 4

by supposing one of them to remain fixed, while the other slides in the direction of its own length through a distance of $2e$, as shown in Fig. 3 or Fig. 4. Fig. 3, in fact, represents a shear combined with right-handed rotation, and Fig. 4 a shear combined with left-handed rotation, as appears by comparing these figures with Fig. 1, which represents shear without rotation.

60. The square and rhombus in these three figures may be regarded as sections of a prism whose edges are perpendicular to the plane of the paper, and figures 3 and 4 show that (neglecting rotation) a shear consists in the relative sliding of parallel planes without change of dis-

tance, the amount of this sliding being proportional to the distance, and being in fact equal to the product of the distance by the numerical measure of the shear. A good illustration of a shear is obtained by taking a book, and making its leaves slide one upon another.

It may be well to remark, by way of caution, that the selection of the planes is not arbitrary as far as direction is concerned. The only planes which are affected in the manner here described are the two sets of planes which make angles of 45° with the axes of the shear (these axes being identical with the diagonals in Fig. 1).

61. Having thus defined and explained the term "shear," which it will be observed denotes a particular species of *strain*, we now proceed to define a shearing stress.

A *shearing stress* may be defined as the combination of two longitudinal stresses at right angles to each other, these stresses being opposite in sign and equal in magnitude; in other words, it consists of a pull in one direction, combined with an equal thrust in a perpendicular direction.

62. Let P denote the intensity of each of these longitudinal stresses; we shall proceed to calculate the stress upon a plane inclined at 45° to the planes of these stresses. Consider a unit cube so taken that the pull is perpendicular to two of its faces, AB and DC (Fig. 5), and the thrust is perpendicular to two other faces, AD , BC . The forces which hold the

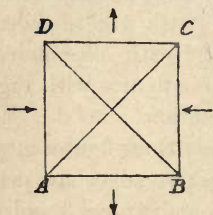


Fig 5.

half-cube ABC in equilibrium are—

(1) An outward force P , uniformly distributed over the face AB , and having for its resultant a single force P acting outward applied at the middle point of AB .

(2) An inward force P , having for its resultant a single force P acting inwards at the middle point of BC .

(3) A force applied to the face AC .

To determine this third force, observe that the other two forces meet in a point, namely, the middle point of AC , that their components perpendicular to AC destroy one another, and that their components along AC , or rather along CA , have each the magnitude $\frac{P}{\sqrt{2}}$; hence their resultant is a force $P\sqrt{2}$, tending from C towards A . The force (3) must be equal and opposite to this. Hence each of the two half-cubes ABC , ADC exerts upon the other a force $P\sqrt{2}$, which is tangential to their plane of separation. The stress upon the diagonal plane AC is therefore a purely tangential stress. To compute its intensity we must divide its amount $P\sqrt{2}$ by the area of the plane, which is $\sqrt{2}$, and we obtain the quotient P . Similar reasoning applies to the other diagonal plane BD . P is taken as the *measure of the shearing stress*. The above discussion shows that it may be defined as the *intensity of the stress either on the planes of purely normal stress, or on the planes of purely tangential stress*.

63. A shearing stress, if applied to a body which has the same properties in all directions (an *isotropic* body), produces a simple shear with the same axes as the stress : for the extension in the direction of the pull will be equal to the compression in the direction of the thrust ; and in the third direction, which is perpendicular to both

of these, there is neither extension nor contraction, since the transverse contraction due to the pull is equal to the transverse extension due to the thrust.

A shearing-stress applied to a body which has not the same properties in all directions produces in general a shear with the same axes as the stress, combined with some other distortion.

In both cases, the quotient of the shearing stress by the shear produced is called the *resistance to shearing*. In the case of an isotropic body, it is also called the *simple rigidity*.

64. The following values of the coefficients of elasticity for solids are reduced from those given in my own papers to the Royal Society (see Phil. Trans., 1868, p. 369), by employing the value of g at the place of observation, namely, 981·4.

	Young's Modulus.	Simple Rigidity.	Volume- Elasticity.	Density
Glass, flint, -	$6\cdot03 \times 10^{11}$	$2\cdot40 \times 10^{11}$	$4\cdot15 \times 10^{11}$	2·942
Another specimen	5·74 „	2·35 „	3·47 „	2·935
Brass, drawn, -	$1\cdot075 \times 10^{12}$	3·66 „	8·471
Steel, - -	2·139 „	8·19 „	$1\cdot841 \times 10^{12}$	7·849
Iron, wrought, -	1·963 „	7·69 „	1·456 „	7·677
„ cast, - -	1·349 „	5·32 „	$9\cdot64 \times 10^{11}$	7·235
Copper, - -	1·234 „	4·47 „	$1\cdot684 \times 10^{12}$	8·843

65. The volume-elasticity was not directly observed, but was calculated from the values of “Young’s modulus” and “simple rigidity,” by a formula which is strictly true for bodies which have the same properties in all directions. The contraction of diameter in lateral directions for a body which is stretched by purely longitudinal stress was also calculated by a formula to which the

same remark applies. The ratio of this lateral contraction to the longitudinal extension is called "Poisson's ratio," and the following were its values as thus calculated for the above bodies :—

Glass, flint,.....	·258	Iron, wrought,.....	·275
Another specimen,	·229	„ cast,.....	·267
Steel,.....	·310	Copper,	·378

Kirchhoff found for steel the value ·294, and Clerk Maxwell found for iron ·267. Cornu (Comptes Rendus, August 2, 1869) found for different specimens of glass the values ·225, ·226, ·224, ·257, ·236, ·243, ·250, giving a mean of ·237, and maintains (with many continental writers) that for all isotropic solids (that is, solids having the same properties in all directions) the true value is $\frac{1}{4}$.

66. The following values of Poisson's ratio were found by Mr. A. Mallock (Proc. Roy. Soc., June 19, 1879):—

Steel,.....	·253	Ivory,.....	·50
Brass,.....	·325	India Rubber,.....	·50
Copper,.....	·348	Paraffin,.....	·50
Lead,.....	·375	Plaster of Paris,....	·181
Zinc (rolled),.....	·180	Cardboard,.....	·2
„ (cast),.....	·230	Cork,	·00
Ebonite,.....	·389		

	Radial due to Longitudinal.	Longitudinal due to Radial.	In Cross Section.
Boxwood,.....	·42	·406	
Beechwood,.....	·53	·408	
White Pine,.....	·486	·372	·227

The heading "Radial due to Longitudinal" means that the applied force is longitudinal (that is, parallel to the length of the tree) and that the contraction along a radius of the tree is compared with the longitudinal extension.

For the value of Poisson's ratio in india rubber, Röntgen found $\cdot 457$, Pulfrich $\cdot 45$, and Amagat from $\cdot 499$ to $\cdot 5$. For its value in jelly, made from gelatine and water, $\cdot 5$ was found by E. Wiedemann, and also by Maurer.

67. The following are reduced from Sir W. Thomson's results (Proc. Roy. Soc., May, 1865) the value of g being $981\cdot 4$:—

	Simple Rigidity.			
Brass, three specimens,	4·03	3·48	3·44	} $\times 10^{11}$
Copper, two specimens,	4·40	4·40		

Other specimens of copper in abnormal states gave results ranging from $3\cdot 86 \times 10^{11}$ to $4\cdot 64 \times 10^{11}$.

The following are reduced from Wertheim's results (Ann. de Chim., ser. 3, tom. xxiii.) g being taken as 981 :—

Different Specimens of Glass (Crystal).

Young's Modulus,.....	3·41 to 4·34, mean 3·96	} $\times 10^{11}$
Simple Rigidity,.....	1·26 to 1·66, „ 1·48	
Volume Elasticity,	3·50 to 4·39, „ 3·89	

Different Specimens of Brass.

Young's Modulus,.....	9·48 to 10·44, mean 9·86	} $\times 10^{11}$
Simple Rigidity,.....	3·53 to 3·90, „ 3·67	
Volume Elasticity, ...	10·02 to 10·85, „ 10·43	

68. Savart's experiments on the torsion of brass wire (Ann. de Chim., 1829) lead to the value $3\cdot 61 \times 10^{11}$ for simple rigidity.

Kupffer's values of Young's modulus for nine different specimens of brass range from $7\cdot 96 \times 10^{11}$ to $11\cdot 4 \times 10^{11}$, the value generally increasing with the density.

For a specimen, of density $8\cdot 4465$, the value was $10\cdot 58 \times 10^{11}$.

For a specimen, of density 8.4930, the value was 11.2×10^{11} .

His values of Young's modulus for steel range from 20.2×10^{11} to 21.4×10^{11} .

69. The following are reduced from Rankine's Rules and Tables, pp. 195 and 196, the mean value being adopted where different values are given :—

	Tenacity.		Young's Modulus.
Steel Bars,	7.93×10^9	2.45×10^{12}
Iron Wire,	5.86 ,,	1.745 ,,
Copper Wire,	4.14 ,,	1.172 ,,
Brass Wire,	3.38 ,,	9.81×10^{11}
Lead, Sheet,	2.28×10^8	5.0×10^{10}
Tin, Cast,	3.17 ,,
Zinc,	5.17 ,,
Ash,	1.172×10^9	1.10×10^{11}
Spruce,	8.55×10^8	1.10 ,,
Oak,	1.026×10^9	1.02 ,,
Glass,	6.48×10^8	5.52×10^{11}
Brick and Cement, ..	2.0×10^7

The tenacity of a substance may be defined as the greatest longitudinal stress that does not produce rupture. It would be equal to the product of Young's modulus by the ultimate extension if the law of simple proportionality held up to the breaking point. In most cases it is very much less than this product.

70. Professor Tait's results for the compressibilities of water, mercury, and glass (results of Challenger Expedition), are stated with reference to two different units of stress. The pressures actually applied are expressed in tons per square inch, and the results are

stated in the form of mean compression per atmosphere. Taking g at Edinburgh as 981.5, we have

1 ton per sq. inch = 154.6 megadynes per sq. centim.

1 atmosphere = 1.0142 „ „

Starting from atmospheric pressure, the mean compression of fresh water per atmosphere at t° C. (t being between 0 and 15), for the first ton per sq. inch, and for the first 3 tons per sq. inch, had the values

$(504 - 3.6t + .04t^2) 10^{-7}$ for first ton per in.,

$(478 - 3.7t + .06t^2) 10^{-7}$ for first 3 tons per in.,

and for sea water the values were

$(462 - 3.2t + .04t^2) 10^{-7}$ for first ton per in.,

$(437.5 - 2.95t + .05t^2) 10^{-7}$ for first 3 tons per in.

At 0° C., the mean compression per atmosphere for the first p tons per sq. inch, was

$(520 - 17p + p^2) 10^{-7}$ for fresh water,

$(481 - 21.25p + 2.25p^2) 10^{-7}$ for sea water.

For solutions of common salt at 0° C., containing s parts of salt to 100 of water, the mean compression per atmosphere for the first p tons per sq. inch was

$$\frac{.00186}{36 + p + s}.$$

The temperature of maximum density of water was found to be lowered about 3° C. by 150 atmospheres of additional pressure.

The temperature of minimum compressibility was lowered by increase of pressure. At atmospheric pressure it was inferred to be about 60° C. for fresh water and about 56° C. for sea water.

The compression of mercury per atmosphere was found to be .000 0036, and that of glass .000 0026. Owing to

slipping of the recording indices during relief of pressure all the above values of compressibility may be a little too small.

71. Amagat (Com. Ren. 1889, I., p. 228) found the following values of compressibility per atmosphere:—

	Compressibility.	Poisson's Ratio.
Mercury,	·000 003918	
Glass (common),	2181	·2451
Do. (crystal),	2405	·2499

He quotes Cautone of Palermo as finding, for four cylinders of glass, the values of Poisson's ratio ·246, ·261, ·264, ·256, giving a mean ·257.

72. The following are the results of Cailletet's experiments (Com. Ren. LXXV. p. 77) which were carried to 600 and 700 atmospheres. We have calculated the column headed "Real" by adding ·000 0039 to the column headed "Apparent."

	Density	Temp.	Compression per atmosphere.	
			Apparent.	Real.
Distilled water } deprived of air, } 1		8	·0000451	·0000490
Bisulphide of carbon, ...		8	980	·0001019
...		8	676	·0000715
Alcohol, ·858		9	701	740
...		9	727	766
Petroleum oil, ·865		11	828	·0000867
Essence of petroleum, ·720		10·5	·0000981	·0001020
Ether,		10	·0001440	1479
Sulphurous acid,		-14	·0003014	3053

73. The following values for compression of liquids are reduced from Grassi results as given by Jamin and Bouty, vol. 1, part 2, p. 131. The value for mercury is probably erroneous.

	Temp. Cent.	Volume- Elasticity.	Compression for one Atmosphere (megadyne per square centim.)
Mercury, - -	0·0	$[3·436 \times 10^{11}]$	$[2·91 \times 10^{-6}]$
Water, - -	0·0	$2·02 \times 10^{10}$	$4·96 \times 10^{-5}$
„ - -	1·5	1·97 „	5·08 „
„ - -	4·1	2·03 „	4·92 „
„ - -	10·8	2·11 „	4·73 „
„ - -	13·4	2·13 „	4·70 „
„ - -	18·0	2·20 „	4·55 „
„ - -	25·0	2·22 „	4·50 „
„ - -	34·5	2·24 „	4·47 „
„ - -	43·0	2·29 „	4·36 „
„ - -	53·0	2·30 „	4·35 „
Ether, - -	{ 0·0 }	$9·2 \times 10^9$	$1·09 \times 10^{-4}$
	{ 0·0 }	7·8 „	1·29 „
	{ 14·0 }	7·2 „	1·38 „
Alcohol, - -	{ 7·3 }	$1·22 \times 10^{10}$	$8·17 \times 10^{-5}$
	{ 13·1 }	1·12 „	8·91 „

According to experiments by Quincke (Berlin Transactions, April 5, 1885) the following are the compressions due to the pressure of one atmosphere. They are expressed in millionths of the original volume:—

	Compression in millionths.		
	at 0° C.	at t° C.	t .
Glycerine,	25·24	25·10	19·00
Rape oil (rüböl),	48·02	58·18	17·80
Almond oil,	48·21	56·30	19·68
Olive oil,	48·59	61·74	18·3
Water,	50·30	45·63	22·93
Bisulphide of carbon,	53·92	63·78	17·00
Oil of turpentine,	58·17	77·93	18·56
Benzol from benzoic acid, —	—	66·10	16·78
Benzol,	—	62·84	16·08
Petroleum,	64·99	74·50	19·23
Alcohol,	82·82	95·95	17·51
Ether,	115·57	147·72	21·36

The coefficient of volume elasticity in C.G.S. units can be calculated from the compression per atmosphere by dividing this compression into 1.014×10^6 . Thus we have approximately :—

	Compression per atmosphere.	Elasticity of volume.
Water,.....	5×10^{-5}	2.03×10^{10}
Mercury,.....	3.9×10^{-6}	2.60×10^{11}
Glass,.....	2.6×10^{-6}	3.90×10^{11}

Viscosity.

74. The frictional resistance which a fluid offers to change of shape—in other words to “shearing”—is found to be directly proportional to the rapidity with which the change is effected—in other words to the “shear per unit time.” Shearing may be regarded as the relative sliding of parallel planes without change of their mutual distance ; and the tangential force per unit area of one of these planes (or the intensity of the shearing stress) is the proper measure of the frictional resistance of the fluid at the actual rate of shearing. The quotient

$$\frac{\text{tangential force per unit area}}{\text{shear per unit time}}$$

is accordingly the proper measure of the quality of the fluid in virtue of which it resists distortion. It is called the *coefficient of viscosity*, or simply the *viscosity*, of the fluid. The omission of the words “per unit time” from the denominator would convert the definition into that of “simple rigidity” (§ 63).

The dimensions of “tangential force per unit area” are $\frac{M}{LT^2}$, and of “shear per unit time,” $\frac{1}{T}$; hence the

dimensions of viscosity are $\frac{M}{LT}$. In the C.G.S. system the coefficient of viscosity denotes the number of dynes per square centim. necessary to produce unit shear per second.

75. When a fluid is forced through a long and very narrow horizontal tube, calculation shows that, if the velocity at the circumference is zero, the volume of fluid which enters or leaves the tube per unit time is

$$\frac{\pi a^4}{8\eta} \frac{p_1 - p_2}{l},$$

l denoting the length of the tube, a its radius, p_1 and p_2 the pressures at its ends, and η the viscosity. In the case of a gas the "volume" is to be reckoned at the pressure $\frac{1}{2}(p_1 + p_2)$. If the velocity at the circumference (called the velocity of *slipping*) is not zero, but is proportional to the tangential force per unit area at the circumference, the expression for the volume per unit time will be

$$\frac{\pi a^3}{2} \frac{p_1 - p_2}{l} \left(\frac{a}{4\eta} + \frac{1}{\beta} \right),$$

where β denotes $\frac{\text{tangential force per unit area}}{\text{velocity of slipping}}$.

This reduces to the previous formula when $\frac{1}{\beta}$ is put equal to zero. Experiment confirms the three laws expressed by the first formula; that is to say, the flow is directly as the difference of pressures at the ends, inversely as the length, and directly as the fourth power of the radius. Some investigators, by experiments of a different kind, have arrived at the conclusion that $\frac{1}{\beta}$ has

a finite and sensible value, but their results are disputed.

76. Elevation of temperature increases viscosity in the case of gases and diminishes it in the case of liquids. The pressure of a gas has no influence on its viscosity (within ordinary ranges of pressure).

The following determinations of viscosity have been mainly selected from a voluminous collection kindly placed at my disposal by Mr. Carl Barus, of the U.S. Geological Survey.

Viscosity of Liquids.

77. The following values for water are based on the results obtained by Sprung (1876), O. E. Meyer (1877), and Slotte (1883):—

Temp.	Viscosity.	Temp.	Viscosity.	Temp.	Viscosity.
0	·0181	25	·0091	50	·0056
5	154	30	81	60	47
10	133	35	73	80	36
15	116	40	67	90	32
20	102	45	61		

Meyer adopts the formula $\frac{·0183}{1 + ·0369 \theta}$, and Slotte the

formula $\frac{·5212}{26 + \theta} - ·00131$,

as approximately representing their results, θ denoting temperature Cent.

Koch (1881) obtained the following values for mercury (omitting superfluous decimals):—

Temp.	Viscosity.	Temp.	Viscosity.	Temp.	Viscosity.
0	·0169	99	·0123	249	·00964
10	162	154	109	315	918
18	156	197	102	340	897

78. Wijkander (1879) gives the following values for other liquids :—

Alcohol.		Ether.		Benzine.	
10	·00156	10	·00283	10	·00746
15	141	15	271	20	641
20	127	20	258	30	555
25	114	25	245	40	488
30	104	30	233	50	433
40	86				
50	72				

Carbon Disulphide.		Chloroform.	
12	·00393	12	·00617
20	370	20	567
25	357	25	539
30	344	30	513
35	332	35	489
		40	467

79. Schöttner found for glycerine the values 42 at 3°, and 8 at 20°.

Obermayer (1877) found for pitch the values 2.1×10^9 between 6° and 7°, 5.3×10^8 at 10°·1, 2.6×10^8 at 12°·2; and for storax the value 13×10^{10} between 15° and 16°.

Carl Barus (1890) has found for marine glue at about 25° the value 2×10^8 ; and for paraffin at about 25° a value exceeding 2×10^{11} .

Viscosity of Gases.

80. Schumann (1884) found for air (omitting superfluous decimals), by vibration experiments :—

Temp.	Viscosity.	Temp.	Viscosity.	Temp.	Viscosity.
0	·000168	40	·000190	80	·000215
10	172	50	196	90	222
20	178	60	202	100	229
30	184	70	209		

Tomlinson (Phil. Trans. 1886) found ·000177 as the value at 12° from a great variety of vibration experiments.

Obermayer (1876) gives as the result of his transpiration experiments the approximate formula

$$\cdot 0001683 (1 + \cdot 00274 \theta),$$

or the nearly equivalent one

$$\cdot 001683 (1 + \alpha \theta)^{.76},$$

θ denoting temperature, and α the coefficient of expansion of air.

81. Carl Barus (1889) from experiments ranging from 418° to 1216° infers, for air and for hydrogen, that the viscosity varies as $(1 + \alpha \theta)^{\frac{3}{2}}$.

Obermayer (1876) gives the following values at 0° for various gases.

Air, . . .	$\cdot 000168$	Nitric oxide, . .	$\cdot 000164$
Oxygen, . .	$\cdot 000187$	Nitrous oxide, .	141
Nitrogen, .	$\cdot 000163$	Carbonic oxide, .	163
Hydrogen, .	$\cdot 0000822$	Carbonic acid, .	141

O. E. Meyer (1873) deduces the following values at 20° from Graham's transpiration experiments.

Air, . . .	$\cdot 000190$	Sulphurous acid, .	$\cdot 000138$
Oxygen, . .	212	Nitric oxide, .	186
Nitrogen, . .	184	Nitrous oxide, .	160
Hydrogen, . .	093	Carbonic oxide, .	184
Steam, . . .	097	Carbonic acid, .	160
Chlorine, . .	147	Marsh gas, .	120
Sulphuretted hy- } drogen, . . }	130	Olefiant gas, .	109
		Ammonia, .	108

CHAPTER VI.

ASTRONOMY.

Size and Figure of the Earth.

82. ACCORDING to the latest determination (Geodesy, by Col. A. R. Clarke, 1880), the semiaxes of the ellipsoid which most nearly agrees with the actual earth are, in feet,

$$a = 20926629, b = 20925105, c = 20854477,$$

which, reduced to centimetres, are

$a = 6.37836 \times 10^8$, $b = 6.37790 \times 10^8$, $c = 6.35639 \times 10^8$, giving a mean radius of 6.37090×10^8 , and a volume of 1.0832×10^{27} cubic centims.

The ellipticities of the two principal meridians (defined as difference divided by half-sum of axes) are

$$\frac{1}{289.5} \text{ and } \frac{1}{295.8}.$$

The longitude of the greatest axis is $8^\circ 15' \text{ W.}$

The ellipsoid of revolution which most nearly represents the actual earth has for its major and minor semiaxes, in feet,

$$a = 20926202, c = 20854895,$$

or, in centimetres,

$$a = 6.37825 \times 10^8, c = 6.35651 \times 10^8;$$

the ratio of c to a being $292.465.: 293.465.$

The average length of one ten-millionth of a quadrant of a meridian is 39·377786 inches, whereas a legal metre is 39·370432 inches. The difference is 1 part in 5354. Hence the mean length of a quadrant of the meridian is $1\cdot00019 \times 10^9$ centims.

The lengths of a degree of latitude and longitude, in centims., in latitude ϕ , are respectively

$$(1111\cdot317 - 5\cdot688 \cos \phi) 10^4,$$

and $(1114\cdot164 \cos \phi - \cdot950 \cos 3 \phi) 10^4$.

83. The mass of the earth, assuming Baily's value 5·67 for the mean density, is $6\cdot14 \times 10^{27}$ grammes. With the value 5·56 obtained by Baille and Cornu (Com. Ren., 1878) for the mean density, the mass is $6\cdot02 \times 10^{27}$.

Day and Year.

Sidereal day,.....	86164	mean solar seconds.
Sidereal year,.....	31,558,150	,,
Tropical year,.....	31,556,929	,,
Angular velocity of earth's rotation,	$\frac{2\pi}{86164} = \frac{1}{13713}$	
Velocity of points on the equator due to earth's rotation,	} 46510	centim. per second.
Velocity of earth in orbit, about		
Centrifugal force at equator due to earth's rotation,.....	} 3·3918	dynes per gramme.

Attraction in Astronomy.

84. The mass of the moon is about $\frac{1}{80}$ of the earth's mass.

The mean distance of the centres of gravity of the earth and moon is 60·2734 equatorial radii of the earth—that is, $3\cdot8444 \times 10^{10}$ centims.

The mean horizontal parallax of the sun is about 8"·8;

hence his mean distance is about 1.493×10^{13} centims., or 92.8 million miles.

The intensity of centrifugal force due to the earth's motion in its orbit (regarded as circular) is $\left(\frac{2\pi}{T}\right)^2 r$, r denoting the mean distance, and T the length of the sidereal year expressed in seconds. This is equal to the acceleration due to the sun's attraction at this distance. Putting for r and T their values, 1.493×10^{13} and 3.1558×10^7 , we have

$$\left(\frac{2\pi}{T}\right)^2 r = .5918.$$

This is about $\frac{1}{1660}$ of the value of g at the earth's surface.

The intensity of the earth's attraction at the mean distance of the moon is about

$$\frac{981}{(60.27)^2} \text{ or } .2701.$$

This is less than the intensity of the sun's attraction upon the earth and moon, which is .5918 as just found. Hence the moon's path is always concave towards the sun.

85. The mutual attractive force F between two masses m and m' , at distance l , is

$$F = C \frac{mm'}{l^2}$$

where C is a constant. To determine its value, consider the case of a gramme at the earth's surface, attracted by the earth. Then we have

$F = 981$, $m = 1$, $m' = 6.14 \times 10^{27}$, $l = 6.37 \times 10^8$; whence we find

$$C = \frac{6.48}{10^8} = \frac{1}{1.543 \times 10^7}.$$

To find the mass m which, at the distance of 1 centim. from an equal mass, would attract it with a force of 1 dyne, we have $1 = Cm^2$;

whence $m = \sqrt{\frac{1}{C}} = 3928$ grammes.

86. To find the acceleration α produced at the distance of l centims. by the attraction of a mass of m grammes, we have

$$\alpha = \frac{F}{m} = C \frac{m}{l^2},$$

where C has the value 6.48×10^{-8} as above.

To find the dimensions of C we have $C = \frac{l^2 \alpha}{m}$, where the dimensions of α are LT^{-2} .

The dimensions of C are therefore

$$L^2 M^{-1} L T^{-2}; \text{ that is, } L^3 M^{-1} T^{-2}.$$

87. The equation $\alpha = C \frac{m}{l^2}$ shows that when $\alpha = 1$ and

$l = 1$, m must equal $\frac{1}{C}$; that is to say, the mass which produces unit acceleration at the distance of 1 centimetre is 1.543×10^7 grammes. If this were taken as the unit of mass, the centimetre and second being retained as the units of length and time, the acceleration produced by the attraction of any mass at any distance would be simply the quotient of the mass by the square of the distance.

It is thus theoretically possible to base a general system of units upon two fundamental units alone; one of the three fundamental units which we have hitherto employed being eliminated by means of the equation

$$\text{mass} = \text{acceleration} \times (\text{distance})^2,$$

which gives for the dimensions of M the expression $L^3 T^{-2}$.

Such a system would be eminently convenient in astronomy, but could not be applied with accuracy to ordinary terrestrial purposes, because we can only roughly compare the earth's mass with the masses which we weigh in our balances.

88. The mass of the earth on this system is the product of the acceleration due to gravity at the earth's surface, and the square of the earth's radius. This product is

$$981 \times (6.37 \times 10^8)^2 = 3.98 \times 10^{20},$$

and is independent of determinations of the earth's density.

The new unit of force will be the force which, acting upon the new unit of mass, produces unit acceleration. It will therefore be equal to 1.543×10^7 dynes; and its dimensions will be

$$\begin{aligned} \text{mass} \times \text{acceleration} &= (\text{acceleration})^2 \times (\text{distance})^2 \\ &= L^4 T^{-4}. \end{aligned}$$

89. If we adopt a new unit of length equal to l centims., and a new unit of time equal to t seconds, while we define the unit mass as that which produces unit acceleration at unit distance, the unit mass will be

$$l^3 t^{-2} \times 1.543 \times 10^7 \text{ grammes.}$$

If we make l the wave-length of the line F in vacuo, say,

$$4.86 \times 10^{-5},$$

and t the period of vibration of the same ray, so that $\frac{l}{t}$ is the velocity of light in vacuo, say,

$$3 \times 10^{10},$$

the value of $l^3 t^{-2}$ or $l \left(\frac{l}{t} \right)^2$ is

$$4.374 \times 10^{16},$$

and the unit mass will be the product of this quantity

into 1.543×10^7 grammes. This product is 6.75×10^{23} grammes.

The mass of the earth in terms of this unit is

$$3.98 \times 10^{20} \div (4.374 \times 10^{16}) = 9100,$$

and is independent of determinations of the earth's density.

CHAPTER VII.

VELOCITY OF SOUND.

90. THE propagation of sound through any medium is due to the elasticity of the medium; and the general formula for the velocity of propagation s is

$$s = \sqrt{\frac{E}{D}},$$

where D denotes the density of the medium, and E the coefficient of elasticity.

91. For air, or any gas, we are to understand by E the quotient

$$\frac{\text{increment of pressure}}{\text{corresponding compression}};$$

that is to say, if P , $P + p$ be the initial and final pressures, and V , $V - v$ the initial and final volumes, p and v being small in comparison with P and V , we have

$$E = \frac{p}{v/V} = p \frac{V}{v}.$$

If the compression took place at constant temperature, we should have

$$\frac{p}{P} = \frac{v}{V} \text{ and } E = P.$$

But in the propagation of sound, the compression is

effected so rapidly that there is not time for any sensible part of the heat of compression to escape, and we have

$$\frac{p}{P} = \gamma \frac{v}{V}, \quad E = \gamma P, \quad s = \sqrt{\gamma \frac{P}{D}},$$

where $\gamma = 1.41$ for dry air, oxygen, nitrogen, or hydrogen.

The value of $\frac{P}{D}$ for dry air at t° Cent. (see p. 44) is

$$(1 + .00366t) \times 7.838 \times 10^8.$$

Hence the velocity of sound through dry air is

$$\begin{aligned} s &= 10^4 \sqrt{1.41 \times (1 + .00366t) \times 7.838} \\ &= 33240 \sqrt{1 + .00366t}; \end{aligned}$$

or approximately, for atmospheric temperatures,

$$s = 33240 + 60t.$$

92. The following are the principal experimental determinations of the velocity of sound in air, reduced to 0° C. :—

	Metres per sec.
Académie des Sciences (1738),.....	332
Benzenberg (1811),	{ 333.7
	{ 332.3
Goldingham (1821),.....	331.1
Bureau des Longitudes (1822),.....	330.6
Moll, van Beek, and Kuytenbrouwer (1822), ...	332.2
Stampfer and Myrback,	332.4
Bravais and Martins (1844),	332.4
Wertheim,	331.6
Stone (1871),.....	332.4
Le Roux,	330.7
Regnault,.....	330.7

93. In the case of any liquid, E denotes the elasticity of volume.*

* See foot note in following page.

For water at $8^{\circ}1$ C. (the temperature of the Lake of Geneva in Colladon's experiment) we have

$$E = 2.08 \times 10^{10}, D = 1 \text{ sensibly};$$

$$\therefore \sqrt{\frac{E}{D}} = \sqrt{E} = 144000:$$

the velocity as determined by Colladon was 143500.

94. For the propagation of sound along a solid, in the form of a thin rod, wire, or pipe, which is free to expand or contract laterally, E must be taken as denoting Young's modulus of elasticity.* The values of E and D will be different for different specimens of the same material. Employing the values given in the Table (§ 64), we have

	Values of E .	Values of D .	Values of $\sqrt{\frac{E}{D}}$, or velocity.
Glass, first specimen, -	6.03×10^{11}	2.942	4.53×10^5
„ second specimen,	5.74 „	2.935	4.42 „
Brass, - - -	1.075×10^{12}	8.471	3.56 „
Steel, - - -	2.139 „	7.849	5.22 „
Iron, wrought, - -	1.963 „	7.677	5.06 „
„ cast, - - -	1.349 „	7.235	4.32 „
Copper, - - -	1.234 „	8.843	3.74 „

95. If the density of a specimen of red pine be $\cdot 5$, and its modulus of longitudinal elasticity be 1.6×10^6 pounds per square inch at a place where g is 981, compute the velocity of sound in the longitudinal direction.

* Strictly speaking, E should be taken as denoting the elasticity for sudden applications of stress—so sudden that there is not time for changes of temperature produced by the stress to be sensibly diminished by conduction. This remark applies to both §§ 93 and 94. For the amount of these changes of temperature, see a later section under Heat.

By the table of stress, at the beginning of this volume, a pound per square inch (g being 981) is 6.9×10^4 dynes per square centim. Hence we have, for the required velocity,

$$\sqrt{\frac{E}{D}} = \sqrt{\frac{1.6 \times 10^6 \times 6.9 \times 10^4}{.5}} = 4.7 \times 10^5$$

centims. per second.

96. The following numbers, multiplied by 10^5 , are the velocities of sound through the principal metals, as determined by Wertheim :—

	At 20° C.	At 100° C.	At 200° C.
Lead, - - -	1.23	1.20	...
Gold, - - -	1.74	1.72	1.73
Silver, - - -	2.61	2.64	2.48
Copper, - - -	3.56	3.29	2.95
Platinum, - -	2.69	2.57	2.46
Iron, - - -	5.13	5.30	4.72
Iron Wire (ordinary),	4.92	5.10	...
Cast Steel, - -	4.99	4.92	4.79
Steel Wire (English),	4.71	5.24	5.00
„ - - -	4.88	5.01	...

The following velocities in wood are from the observations of Wertheim and Chevandier, Comptes Rendus, 1846, pp. 667 and 668 :—

	Along Fibres.	Radial Direction.	Tangential Direction.
Pine, - - -	3.32×10^5	2.83×10^5	1.59×10^5
Beech, - - -	3.34 „	3.67 „	2.83 „
Hornbeam, - -	3.92 „	3.41 „	2.39 „
Birch, - - -	4.42 „	2.14 „	3.03 „
Fir, - - -	4.64 „	2.67 „	1.57 „
Acacia, - - -	4.71 „		
Aspen, - - -	5.08 „		

Musical Strings.

97. Let M denote the mass of a string per unit length,
 F „ stretching force,
 L „ length of the vibrating portion;
 then the velocity with which pulses travel along the string is

$$v = \sqrt{\frac{F}{M}},$$

and the number of vibrations made per second is

$$n = \frac{v}{2L}.$$

Example.

For the four strings of a violin the values of M in grammes per centimetre of length are

$$\cdot 00416, \quad \cdot 00669, \quad \cdot 0106, \quad \cdot 0266.$$

The values of n are

$$660, \quad 440, \quad 293\frac{1}{3}, \quad 195\frac{5}{9};$$

and the common value of L is 33 centims. Hence the values of v or $2Ln$ are

$$43560, \quad 29040, \quad 19360, \quad 12910$$

centims. per second; and the values of F or Mv^2 , in dynes, are

$$7\cdot89 \times 10^6, \quad 5\cdot64 \times 10^6, \quad 3\cdot97 \times 10^6, \quad 4\cdot43 \times 10^6.$$

Faintest Audible Sound.

98. Lord Rayleigh (Proc. R. S., 1877, vol. xxvi. p. 248), from observing the greatest distance at which a whistle giving about 2730 vibrations per second, and

blown by water-power, was audible without effort in the middle of a fine still winter's day, calculates that the maximum velocity of the vibrating particles of air at this distance from the source was $\cdot 0014$ centims. per second, and that the amplitude was $8\cdot 1 \times 10^{-8}$ centims., the calculation being made on the supposition that the sound spreads uniformly in hemispherical waves, and no deduction being made for dissipation, nor for waste energy in blowing.

CHAPTER VIII.

LIGHT.

99. ALL kinds of light are believed to have the same velocity in *vacuo*. The absolute index of refraction for light of given refrangibility in a given medium is equal to the quotient obtained by dividing the velocity of that kind of light in the medium into the velocity of light in *vacuo*. The frequency of vibration (that is the number of vibrations per second) is unchanged when a ray passes out of one medium into another; but the wave-length changes in the inverse ratio of the index of refraction. The product of the wave-length in any medium by the index of refraction for that medium is equal to the wave-length *in vacuo*.

100. The best determinations of the velocity of light are those of Michelson and Newcomb, by the method of the revolving mirror, and of Cornu, by the method of the toothed wheel. The resulting velocity *in vacuo* is about 2.999×10^{10} centims. per sec.

Professor Newcomb remarks that the value 299860 km. per sec. for the velocity of light, combined with Clarke's value 6378.2 km. for the earth's equatorial radius,

and Nyren's value $20''\cdot492$ for the constant of aberration, gives for the solar parallax the value $8''\cdot794$.

The following summary of results for velocity *in vacuo* is given by Professor Newcomb:—

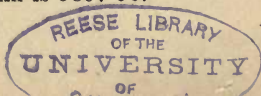
	Km. per sec.
Michelson, at Naval Academy, in 1879,.....	299910
Michelson, at Cleveland, 1882,.....	299853
Newcomb, at Washington, 1882, using only results supposed to be nearly free from constant errors,.....	299860
Newcomb, including all determinations,.....	299810
Foucault, at Paris, in 1862,.....	298000
Cornu, at Paris, in 1874,	298500
Cornu, at Paris, in 1878,.....	300400
This last result as discussed by Helmholtz,	299990
Young and Forbes, 1880-81,.....	301382

101. Wave-lengths are usually stated in *tenth-metres*, a tenth-metre being 10^{-10} of a metre, or 10^{-8} of a centim. Thus 5893 tenth-metres = $5\cdot893 \times 10^{-5}$ centims.

The following wave-lengths are adopted by Rowland and Bell (May, 1888), from their own observations, for air at 20° C. and 760 mm. (at Baltimore), and are probably the best determinations yet made:—

A (line between head and tail of group),	7621·31
B " " "	6884·11
C	6563·07
D ₁	5896·18
D ₂	5890·22
E ₁	5270·52
E ₂	5269·84
b ₁	5183·82
F	4861·51

Their result for D₁ reduced to vacuum is $5897\cdot90$.



They give the following list of results obtained by various observers for D_1 :—

Mascart,.....	5894·3	Ångström corr. by Thalen,.....	5895·89
Van der Willigen,.....	5898·6	Müller and Kempf,.....	5896·25
Ångström,.....	5895·13	Macé de Lépinay,.....	5896·04
Ditscheiner,.....	5897·4	Kurlbaum,.....	5895·90
Peirce,.....	5896·27	Bell,.....	5896·18

102. The following list of adopted values for the chief lines in the solar spectrum, including the ultra-violet, is given in the new edition of Watts' *Index of Spectra* (1889).

Designation and Origin.	Wave-length in Air.	Refractive Index of Air.
A	7604·0	1·00029286
B	6867·0	29350
C (H)	6562·1	29383
D (Na) 5892·12	$\left\{ \begin{array}{l} 5895·13 \\ 5889·12 \end{array} \right\}$	29470
E (Ca and Fe)	5269·13	29584
b_1 (Mg)	5183·10	
b_2 (Mg)	5172·16	
b_3 (Ni and Fe)	5168·48	
b_4 (Mg and Fe)	5166·88	
F (H)	4860·72	29685
G (Fe)	4307·25	29873
H (Ca)	3968·1	
K (Ca)	3933·0	30028
L (Fe)	3819·8	300955
M (Fe)	3727·0	301475
N (Fe)	3580·5	30212
O (Fe, double) ..	3439·8	30336
P (Fe and Ti) ..	3359·2	30397
Q (Fe)	3284·9	30459
R (Fe and Ca) ..	3179·0	30555
r (Fe, double) ..	3144·3	3073?
S_1 (Ni, double) ..	3100·6	3100·0
S_2 (Fe, triple) ..	3099·5	
s (Fe)	3046·4	
T (Fe, double) ..	3019·7	
t (Fe)	2994·3	
U (Fe)	2947·8	

For the principal bright line of lithium the wave-length in air is 6706; and for the thallium line 5349.

103. The following wave-lengths were adopted by Ångström for air at 16° C. and 760 mm. pressure (at Upsal), and were long regarded as the standard values. We give in the third column the corresponding frequencies deduced by taking the velocity as 3×10^{10} centims. per sec.

Fraunhofer Line.	Wave-length, tenth-metres.	Vibrations per sec.
A	7604	3.945×10^{14}
B	6867	4.369
C	6562.01	4.572
D (mean).....	5892.12	5.092
E	5269.13	5.693
F	4860.72	6.172
G	4307.25	6.965
H ₁	3968.01	7.560
H ₂	3933.00	7.628

According to Langley (Com. Ren., Jan., 1886), the solar spectrum extends beyond the red as far as wave-length 27000, and the radiation from terrestrial bodies at temperatures below 100° C. extends as far as wave-length 150000 tenth-metres. The corresponding frequencies are 1.1×10^{14} and 2×10^{13} . On the other hand the frequency corresponding to the wave-length 2948 in the extreme ultra-violet is 1.017×10^{15} .

INDICES OF REFRACTION OF SOLIDS.

104. Dr. Hopkinson (Proc. R. S., June 14, 1877) has determined the indices of refraction of the principal varieties of optical glass made by Messrs. Chance, for

the fixed lines A, B, C, D, E, b , F, (G), G, h , H_1 . By D is to be understood the more refrangible of the pair of sodium lines; by b the most refrangible of the group of magnesium lines; by (G) the hydrogen line near G.

In connection with the results of observation, he employs the empirical formula

$$\mu - 1 = a\{1 + bx(1 + cx)\},$$

where x is a numerical name for the definite ray of which μ is the refractive index. In assigning the value of x , four glasses—hard crown, soft crown, light flint, and dense flint—were selected on account of the good accord of their results; and the mean of their indices for any given ray being denoted by $\bar{\mu}$, the value assigned to x for this ray is $\bar{\mu} - \bar{\mu}_F$ where $\bar{\mu}_F$ denotes the value of $\bar{\mu}$ for the line F.

The value of $\bar{\mu}$ as a function of λ , the wave-length in 10^{-4} centimetres, was found to be approximately

$$\begin{aligned}\bar{\mu} = 1.538414 + 0.0067669\frac{1}{\lambda^2} - 0.0001734\frac{1}{\lambda^4} \\ + 0.000023\frac{1}{\lambda^6}.\end{aligned}$$

The following were the results obtained for the different specimens of glass examined:—

Hard Crown, 1st specimen, density 2.48575.

$a = 0.523145$, $b = 1.3077$, $c = -2.33$.

Means of observed values of μ .

A 1.511755;	B 1.513624;	C 1.514571;	D 1.517116;
E 1.520324;	b 1.520962;	F 1.523145;	(G) 1.527996;
G 1.528348;	h 1.530904;	H_1 1.532789.	

Soft Crown, density 2·55035.

$$\alpha = 0\cdot5209904, \quad b = 1\cdot4034, \quad c = -1\cdot58.$$

Means of observed values of μ .

A 1·508956; B 1·510918; C 1·511910; D 1·514580;
E 1·518017; b 1·518678; F 1·520994; (G) 1·526208;
G 1·526592; h 1·529360; H₁ 1·531415.

Extra Light Flint Glass, density 2·86636.

$$\alpha = 0\cdot549123, \quad b = 1\cdot7064, \quad c = -0\cdot198.$$

Means of observed values of μ .

A 1·534067; B 1·536450; C 1·537682;
D 1·541022; E 1·545295; b 1·546169;
F 1·549125; (G) 1·555870; G 1·556375;
h 1·559992; H₁ 1·562760.

Light Flint Glass, density 3·20609.

$$\alpha = 0\cdot583887, \quad b = 1\cdot9605, \quad c = 0\cdot53.$$

Means of observed values of μ .

B 1·568558; C 1·570007; D 1·574013;
E 1·579227; b 1·580273; F 1·583881;
(G) 1·592184; G 1·592825; h 1·597332;
H₁ 1·600717.

Dense Flint, density 3·65865.

$$\alpha = 0\cdot634744, \quad b = 2\cdot2694, \quad c = 1\cdot48.$$

Means of observed values of μ .

B 1·615704; C 1·617477; D 1·622411;
E 1·628882; b 1·630208; F 1·634748;
(G) 1·645268; G 1·646071; h 1·651830;
H₁ 1·656229.

Extra Dense Flint, density 3·88947.

$$\alpha = 0\cdot664226, \quad b = 2\cdot4446, \quad c = 1\cdot87.$$

Means of observed values of μ .

A 1·639143; B 1·642894; C 1·644871;
D 1·650374; E 1·657631; b 1·659108;
F 1·664246; (G) 1·676090; G 1·677020;
h 1·683575; H₁ 1·688590.

Double Extra Dense Flint, density 4·42162.

$a = 0\cdot727237$, $b = 2\cdot7690$, $c = 2\cdot70$.

Means of observed values of μ .

A 1·696531 ;	B 1·701080 ;	C 1·703485 ;
D 1·710224 ;	E 1·719081 ;	b 1·720908 ;
F 1·727257 ;	(G) 1·742058 ;	G 1·743210 ;
h 1·751485.		

105. The following indices of rock salt, sylvin, and alum for the chief Fraunhofer lines are from Stefan's observations :—

	Rock Salt at 17° C.	Sylvin at 20° C.	Alum at 21° C.
A	1·53663	1·48377	1·45057
B	·53918	·48597	·45262
C	·54050	·48713	·45359
D	·54418	·49031	·45601
E	·54901	·49455	·45892
F	·55324	·49830	·46140
G	·56129	·50542	·46563
H	·56823	·51061	·46907

106. Indices of other singly refracting solids :—

	Index of Refraction.	Kind of Light.	Observer.
Diamond,	2·470	D	Schrauf.
Fluorspar,	1·4339	D	Stefan.
Amber,	1·532	D	Kohlrausch.
Rosin,	1·545	Red	Jamin.
Copal,	1·528	Red	„
Gum Arabic,	1·480	Red	„
Peru Balsam,	1·593	D	Baden Powell.
Canada Balsam, ...	1·528	Red	Wollaston.

Effect of Temperature.

According to Stefan, the index of refraction of glass *increases* by about ·000 002 for each degree Cent. of increase of temperature, and the index of rock salt

diminishes by about $\cdot 000\,037$ for each degree of increase of temperature.

107. Doubly refracting crystals :—

Uniaxal Crystals.

	Ordinary Index.	Extraordinary Index.	Kind of Light.	Temp.	Observer.
Apatite,.....	1·64607	1·64172	D	21°	Heusser.
Ice,.....	1·3060	1·3073	Red		Reusch.
Iceland-spar,.....	1·65844	1·48639	D	24°	v. d. Willigen.
Nitrate of Soda,	1·5854	1·3369	D	23°	F. Kohlrausch.
Tourmaline,.....	1·6366	1·6193	D	22°	Heusser.
Tourmaline,.....	1·6479	1·6262	Green	22°	„
Zircon.....	1·92	1·97	Red		de Senarmont.

G. Müller gives the following values for quartz. (The subscript under the figure 0 denotes the number of zeros after the decimal point.)

	Ordinary Index.	Extraordinary Index.
B	1·54108 - $\cdot 0_5432\,t$	1·54784 - $\cdot 0_5457\,t$
C	197 - $402\,t$	875 - $454\,t$
D	432 - $432\,t$	1·55116 - $485\,t$
F	976 - $426\,t$	674 - $462\,t$
G _{γ}	1·55404 - $\cdot 0_5459\,t$	1·56114 - $\cdot 0_5467\,t$

Biaxal Crystals.

THREE PRINCIPAL INDICES OF REFRACTION FOR SODIUM LIGHT.

	Least.	Intermediate.	Greatest.	Temp.	Observer.
Arragonite,	1·53013	1·68157	1·68589		Rudberg.
Borax,.....	1·4463	1·4682	1·4712	23°	Kohlrausch.
Mica,.....	1·5609	1·5941	1·5997	23°	„
Nitre,.....	1·3346	1·5056	1·5064	16°	Schrauf.
Nitrate of Potash, }	1·3346	1·5056	1·5064	16°	„
Selenite,.....	1·52082	1·52287	1·53048	17°	v. Lang.
Sulphur (prismatic) }	1·9505	2·0383	2·2405	16°	Schrauf.
Topaz,.....	1·61161	1·61375	1·62109		Rudberg.

INDICES OF REFRACTION FOR LIQUIDS.

108. The following indices of refraction are taken from van der Willigen's results. d_t denotes density at temp. t° :—

	Carbon Disulphide.	Water.	Benzol.
	$d_{15} =$		$d_{18} =$
Density,	1.2709		0.8775
Temp.,	17°	20°·2	18°
<i>Red</i> A	1.61136	1.32889	1.4879
B	1756	3038	910
C	2086	3113	926
D	3034	3298	1.4972
E	4320	3522	1.5033
—F	5529	3713	089
G	7975	4057	197
<i>Violet</i> H	1.70277	1.34343	1.5295

The following are from Kundt's results :—

	Alcohol.	Ether.	Chloroform.
	$d_{18} =$	$d_{18} =$	$d_{18} =$
Density,	0.800	0.713	1.501
Temp.,	15°	15°	15°
A	1.3596	1.3550	1.4440
<i>red</i> B	609	565	458
C	615	573	467
D	633	594	492
<i>Blue</i> E	656	618	525
F	675	641	554
G	713	681	1.4611
H	1.3745	1.3713	—

INDICES FOR GASES.

109. Indices of refraction of air at 0° C. and 760 mm. for the principal Fraunhofer lines.

	According to Ketteler.	According to Lorenz.
A	1·00029286	1·00028935
B	29350	28993
C	29383	29024
D	29470	29108
E	29584	29217
F	29685	29312
G	29873	29486
H	30026	29631

110. The formula established by the experiments of Biot and Arago for the index of refraction of air at various pressures and temperatures was

$$\mu - 1 = \frac{0002943}{1 + \alpha t} \cdot \frac{h}{760},$$

α denoting the coefficient of expansion 00366, and h the pressure in millims. of mercury at zero. As the pressure of 760 millims. of such mercury at Paris is $1\cdot0136 \times 10^6$ dynes per sq. cm., the general formula applicable to all localities alike will be

$$\mu - 1 = \frac{0002943}{1 + 00366t} \cdot \frac{P}{1\cdot0136 \times 10^6},$$

where P denotes the pressure in dynes per sq. cm. This can be reduced to the form

$$\mu - 1 = \frac{0002903}{1 + 00366t} \cdot \frac{P}{10^6}.$$

111. According to Mascart, $\mu - 1$ for any gas is proportional not to $\frac{h}{1 + \alpha t}$ but to $\frac{h + \beta h^2}{1 + \alpha' t}$.

where β and α' are coefficients which vary from one gas to another. In the following table, the column headed μ_0 contains the indices for 0° and 760 mm. at Paris. The

next column contains the value of β multiplied by 10^7 (it being understood that h is expressed in millimetres), and the next column the value of α' . All these data are for the light of a sodium flame :—

	μ_0	$\beta \times 10^7$	α'
Air,	1·0002927	7·2	·00382
Nitrogen,	2977	8·5	382
Oxygen,	2706	11·1
Hydrogen,	1387	− 8·6	378
Nitrous Oxide,	5159	88	388
Nitrous Gas,	2975	7	367
Carbonic Oxide,	3350	8·9	367
Carbonic Acid,	4544	72	406
Sulphurous Acid,	7036	25	460
Cyanogen,	8216	27·7

More recent observations by Benoît with Fizeau's dilatometer give

$$1\cdot0002923$$

as the index of refraction of air for the line D_2 at 0° C. and 760 mm., and

$$\cdot003667$$

as the temperature coefficient.

Coefficient of Dispersive Power.

112. Assuming Cauchy's formula

$$\mu - 1 = a \left(1 + \frac{b}{\lambda^2} \right)$$

(where λ is the wave-length), which is known to be approximately true for air within the limits of the visible spectrum, the constant b may be called the *coefficient of dispersive power*. Employing as the unit of length for

λ the 10^{-4} of a centimetre, Mascart (Ann. de l'Ecole Normale, 1877, p. 62) has obtained the following values for b :—

Coefficient of Dispersion.	
Air,	·0058
Nitrogen,	·0067
Oxygen,	·0064
Hydrogen,	·0043
Carbonic Oxide,	·0075
Carbonic Acid,	·0052
Nitrous Oxide,	·0125
Cyanogen,	·0100

According to Mascart, the ratio of dispersion to deviation for the two lines B and H is ·024 for air, ·032 for the ordinary ray in quartz, ·038 for light crown glass, ·040 for water, and ·046 for the ordinary ray in Iceland-spar.

Rotation of Plane of Polarization.

113. The rotation produced by 1 millim. of thickness of quartz cut perpendicular to the axis has the following values for different portions of the spectrum, according to the observations of Soret and Sarasin (Com. Ren. t. xcv. p. 635, 1882), the temperature of the quartz being 20° C. :—

A.....	$12^{\circ}\cdot668$	E.....	$27^{\circ}\cdot543$
B	$15^{\circ}\cdot746$	F.....	$32^{\circ}\cdot773$
C	$17^{\circ}\cdot318$	G.....	$42^{\circ}\cdot604$
D ₁	$21^{\circ}\cdot684$	H	$51^{\circ}\cdot193$
D ₂	$21^{\circ}\cdot727$		

According to the same observers, the rotation at t° C. is equal to the rotation at 0° C. multiplied by $1 + \cdot000179 t$.

The rotation (for sodium light) produced by a length of 10 centims. of solution of cane sugar, at 20° C., containing n parts by weight of sugar in 100 of the solution, is $0^{\circ}\cdot665 \times n$. For milk sugar it is $0^{\circ}\cdot5253 \times n$.

Units of Illuminating Power.

114. The British "Candle" is a spermaceti candle, $\frac{1}{8}$ inch in diameter (6 to the lb.), burning 120 grains per hour.

The "Pentane Standard," introduced by Mr. Vernon Harcourt, is intended to represent an average "candle," with the advantage of greater uniformity and a whiter light. It is obtained by burning a mixture of pentane vapour and air from a burner of specified construction. The pentane is a volatile liquid prepared by a process of purification and rectification from American petroleum. When 3 cub. ft. of air and 9 cub. in. of liquid pentane are brought together in a gasholder, the pentane vaporizes and mixes with the air, yielding altogether 4.05 cub. ft. of standard air-gas. This gas is burnt from a $\frac{1}{4}$ in. circular orifice so as to form a steady conical flame $2\frac{1}{2}$ in. high. The rate at which the gas burns is also measured, and must approximate closely to half a cubic foot of air-gas per hour.

The French "Carcel" is a lamp of specified construction, burning 42 grammes of pure Colza oil per hour. One "carcel" is equal to about $9\frac{1}{2}$ "candles."

The unit adopted by the International Congress at Paris, April, 1884, is a square centimetre of molten platinum at the temperature of solidification. The surface illuminated by it in photometric tests is to be

normally opposite to the surface of the molten platinum. According to the experiments of M. Violle, the author of this unit, it is equal to 2·08 carcels. It is therefore about $19\frac{3}{4}$ candles.

At the International Congress of 1890 it was agreed to adopt one twentieth part of this unit as the international standard candle, and to call it the "Decimal Candle" (*bougie décimale*).

The commonest unit in Germany is the Hefner-Alteneck lamp, which, with a cylindrical wick 8 mm. in diameter, burns amyl-acetate, with a flame 40 mm. high. It is about $1\frac{1}{4}$ candle, 15·9 Hefner units being equal to 2·08 carcels, or $19\frac{3}{4}$ British candles, or 20 decimal-candles.

CHAPTER IX.

HEAT.

115. THE primary unit of heat is the quantity of heat which is equivalent to the unit quantity of work. Thus in the C.G.S. system the heat which is equivalent to 1 erg is the primary unit of heat ; or, more briefly (since heat is a form of energy), *the erg is the primary unit of heat in the C.G.S system.*

It is more usual to employ a *secondary unit of heat*, namely, the quantity of heat required to raise unit mass of cold water through 1° of the thermometric scale employed. The dimensions of the secondary unit of heat are $M\Delta$, Δ denoting length of a degree.

Mechanical Equivalent of Heat.

116. The ratio of the secondary to the primary unit of heat is commonly called the "Mechanical Equivalent of Heat," or the "Mechanical Equivalent of the Unit of Heat," or, more briefly, "Joule's Equivalent," and is denoted by the symbol J . It is the number of units of work required to raise the temperature of unit mass of water 1° .

For a more precise definition see § 126.

117. Since the secondary unit of heat varies jointly as the unit of mass and the length of a degree of temperature, the dependence of the numerical value J upon the units employed can be calculated as follows, the length of a degree being denoted by Δ :—

Secondary unit of heat varies as $M\Delta$.

Absolute unit of work „ $ML^2 T^{-2}$.

Gravitation unit of work „ ML .

Hence the numerical value J varies as $\frac{\Delta}{L}$, if gravitation units are employed, and as $\frac{\Delta T^2}{L^2}$ if absolute units are employed ; and the dimensions of the unit in terms of which J is the numerical value are $\frac{L}{\Delta}$ and $\frac{L^2}{\Delta T^2}$ in the two cases respectively.

Ex. 1. In gravitation units,

$$\begin{aligned} 772 \frac{\text{foot}}{\text{deg. Fahr.}} &= 1390 \frac{\text{foot}}{\text{deg. Cent.}} = 424 \frac{\text{metre}}{\text{deg. Cent.}} \\ &= 42400 \frac{\text{centim.}}{\text{deg. C.}}, \end{aligned}$$

the numerical values, 772, 1390, etc., being the values of J in the several systems of reckoning.

Ex. 2. In absolute units, if 25187 be the value of J in the foot-pound-second-Fahrenheit system, what is its value in the C.G.S. system ?

We have

$$25187 \frac{(\text{foot})^2}{\text{deg. F.}} = x \frac{(\text{centim.})^2}{\text{deg. C.}};$$

whence

$$x = 25187 \times (30.48)^2 \times \frac{9}{5} = 4.212 \times 10^7.$$

To reduce from gravitational to absolute units, the

value of J must be multiplied by the value of g . For example, in the C.G.S. system, if 42400 be taken as the value of J in gravitation units, and 981 as the value of g , the value of J in absolute units will be $42400 \times 981 = 4.159 \times 10^7$ nearly.

118. The *mean thermal capacity* of a body *between two stated temperatures* is defined as the quantity of heat required to raise it from the lower temperature to the higher, divided by the difference of the temperatures. The *thermal capacity at a stated temperature* is the limit to which the mean thermal capacity tends when the two temperatures approach the stated temperature; that is, it is the value of the differential coefficient $\frac{dQ}{dt}$, Q denoting the quantity of heat given, and t the temperature produced.

The thermal capacity of unit mass of any substance is called the *specific heat* of that substance.

119. Rowland has directly determined the specific heat of water at temperatures from 5° to 36° by a

Temp. C.		Temp. C.		Temp. C.	
5°	4.212×10^7	16°	4.187×10^7	27°	4.171×10^7
6	4.209 "	17	.185 "	28	.171 "
7	4.207 "	18	.183 "	29	.170 "
8	4.204 "	19	.181 "	30	.171 "
9	4.202 "	20	.179 "	31	.171 "
10	4.200 "	21	.177 "	32	.171 "
11	4.198 "	22	.176 "	33	.172 "
12	4.196 "	23	.175 "	34	.172 "
13	4.194 "	24	.174 "	35	.173 "
14	4.192 "	25	.173 "	36	.173 "
15	4.189 "	26	.172 "		

modification of Joule's method of raising the temperature of water by stirring it; and his final results for the number of ergs required to raise the temperature of a gramme of water through 1° C. of the absolute thermodynamic scale are given in the annexed table.

120. The following are the corresponding gravitation values, (1) for Baltimore ($g = 980.0$), where the experiments were made, (2) for Manchester ($g = 981.3$), where Joule's experiments were made. The values for Baltimore are given in metres per degree Cent. Those for Manchester are given both in met. per deg. Cent. and in feet per degree Fahr., so as to be comparable with Joule's well-known result, 772.

Temp. C.	Baltimore.	Manchester.		Temp. C.	Baltimore.	Manchester.	
5°	429.8	429.2	782.3	21°	426.2	425.7	775.9
6	429.5	428.9	781.8	22	426.1	425.6	775.7
7	429.3	428.7	781.4	23	426.0	425.5	775.5
8	429.0	428.4	780.9	24	425.9	425.4	775.3
9	428.8	428.2	780.5	25	425.8	425.3	775.1
10	428.5	428.0	780.1	26	425.7	425.2	774.9
11	428.3	427.8	779.7	27	425.6	425.0	774.7
12	428.1	427.6	779.4	28	425.6	425.0	774.7
13	427.9	427.4	779.0	29	425.5	424.9	774.5
14	427.7	427.2	778.6	30	425.6	425.0	774.7
15	427.4	426.9	778.1	31	425.6	425.0	774.7
16	427.2	426.7	777.7	32	425.6	425.0	774.7
17	427.0	426.5	777.3	33	425.7	425.2	774.9
18	426.8	426.3	777.0	34	425.7	425.2	774.9
19	426.6	426.1	776.6	35	425.8	425.3	775.1
20	426.4	425.9	776.2	36	425.8	425.3	775.1

It thus appears from Rowland's experiments that the round number 42 millions is the true value of J in ergs per gmi. deg., at the temperature 10° C. or 50° F., and

that, at this temperature, the gravitation values of J at Manchester are (to the nearest integer)

428 kgm. metres per kgm. deg. C.,

780 ft. lbs. per lb. deg. F.

1404 ft. lbs. per lb. deg. C.,

121. An inspection of the Table in § 119 shows that the specific heat of water does not regularly increase with the temperature, as was formerly believed, but first decreases and then increases, attaining its minimum at about 29° C.

When temperatures are expressed in degrees of a mercurial thermometer with its tube divided into parts of equal volume, the results will vary according to the kind of glass of which the thermometer is made. Rowland gives results for four different standard mercurial thermometers, and they place the minimum at points ranging from 18° or 19° (Geissler thermometer) to 29° (Kew thermometer).

On the other hand Regnault's determinations make the specific heat of water at t° proportional to

$$1 + \cdot 00004t + \cdot 0000009t^2$$

and the mean specific heat between 0° and t° proportional to

$$1 + \cdot 00002t + \cdot 0000003t^2.$$

Rowland remarks, "The principal experiments on the subject were published by Regnault in 1850, and these have been accepted to the present time. It is unfortunate that these experiments were all made by mixing water above 100° with water at ordinary temperatures, *it being assumed that water at ordinary temperatures changed little, if any.* An interpolation formula was

then found to represent the results ; and *it was assumed* that the same formula held at ordinary temperature, or even as low as 0°C . It is true that Regnault experimented on the subject at points around 4°C . by determining the specific heat of lead in water at various temperatures ; but the results were not of sufficient accuracy to warrant any conclusions except that the variation was not great."

122. The relation between heat and work originally deduced by Joule from his experiments on the stirring of water was 772 foot-pounds of work (at Manchester) for as much heat as raises a pound of water through 1°Fahr . This is 1389.6 foot-pounds for a pound of water raised 1°C ., or 1389.6 foot-grammes for a gramme of water raised 1°C . As a foot is 30.48 centims., and the value of g at Manchester is 981.3, this is $1389.6 \times 30.48 \times 981.3$, that is 4.156×10^7 ergs, for 1 gramme of water raised 1°C .

A later experiment on the heat produced by the friction of water is described by Joule in Phil. Trans., 1878, Part II., leading to the result that the work required to raise a pound of water from 60° to 61°Fahr . is 772.55 foot-pounds at sea-level and the latitude of Greenwich. The number of ergs required to raise a gramme of water through 1°C . at the temperature of 60.5°Fahr . or 15.8°C . would accordingly be

$$772.55 \times \frac{9}{5} \times 30.48 \times 981.17 = 4.159 \times 10^7.$$

Rowland's result for this temperature ($15\frac{5}{8}^{\circ}\text{C}$.) is 4.187×10^7 .

In the British Association Report for 1867 (and "Reprint of Reports on Electrical Standards," p. 186)

is given a determination by Joule based on the heating of a conductor by an electric current. The result given is 25187 foot-grain-second (absolute) units of work for a grain of water raised 1° Fahr. The corresponding number of ergs for a gramme of water raised 1° C. is

$$25187 \times \frac{9}{5} \times (30\cdot48)^2 = 4\cdot212 \times 10^7.$$

In view of the fact that the electrical resistance was expressed in terms of the B.A. standard, which is now known to be only $\cdot98656$ of a true ohm, the above result must be multiplied by $\cdot98656$, giving $4\cdot155 \times 10^7$. The average temperature of the water was $18^{\circ}\cdot63$ C.

The principal part of the difference between Joule's results and Rowland's has been traced to want of agreement between the mercurial thermometers employed by Joule and the perfect air-thermometer.

Comparison of Mercurial with Air Thermometer.

123. Rowland, at p. 89 of his paper on the Mechanical Equivalent of Heat, gives for the excess of the reading T of a mercurial thermometer above the reading t of an air thermometer, the formula

$$T - t = at(100 - t)(b - t);$$

and gives as a specimen of actual values

$$a = \cdot00000033, \quad b = 290.$$

The observed excess $T - t$ had in this instance the following values between 0° and 100° :—

Temp.,	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°
Excess,	0	$\cdot08$	$\cdot14$	$\cdot18$	$\cdot20$	$\cdot20$	$\cdot18$	$\cdot14$	$\cdot10$	$\cdot05$	0

P. Chappuis gives, in vol. VI. of "Travaux et Memoires du Bureau International," p. 116, the follow-

ing differences between a hydrogen thermometer and a mercury thermometer of hard glass by Tonnelot :—

Mercury Thermometer.	Mercury minus Hydrogen.	Mercury Thermometer.	Mercury minus Hydrogen.
— 20	— ·172	50	·103
— 10	— ·073	60	·090
0	0	70	·072
10	+ ·052	80	·050
20	·085	90	·026
30	·102	100	0
40	·107		

Comparison with the Absolute Thermo-dynamic Scale.

124. The absolute thermodynamic scale of temperature is defined by the statement that, when a perfect thermodynamic engine takes in heat at one temperature and gives out heat at another, the quantities of heat which it takes in and gives out are proportional to these temperatures. In the following comparisons, the constant which expresses the difference between the zero of the absolute scale and that of the ordinary centigrade scale is neglected.

Rowland gives, at p. 114 of his paper on the Mechanical Equivalent of Heat, the following table of corrections for reducing temperatures by air thermometer (at constant volume) to the absolute scale :—

Air Ther.	Correction.	Air Ther.	Correction.
0	0	80	— ·0044
10	— ·0028	90	— ·0022
20	— ·0048	100	0
30	— ·0061	200	+ ·037
40	— ·0067	300	+ ·092
50	— ·0068	400	+ ·157
60	— ·0063	500	+ ·228
70	— ·0054		

These corrections were calculated by the formula

$$- \cdot 000\ 88\ t \frac{100 - t}{273 + t},$$

t denoting temperature by air thermometer.

To reduce his Kew standard thermometer to the absolute scale, he employs the following corrections:—

Temp.	Correction.	Temp.	Correction.
0	0	60	-·06
10	-·03	70	-·04
20	-·05	80	-·02
30	-·06	90	-·01
40	-·07	100	0
50	-·07		

Relations between various Units of Heat.

125. The secondary unit of heat employed in the C.G.S. system is the quantity of heat required to raise a gramme of cold water 1° C. It is called for brevity the *gramme-degree*. No definite temperature has been agreed upon for the cold water. We shall adopt $4\cdot2 \times 10^7$, or 42 millions, as the number of ergs equivalent to the gramme-degree. This, according to Rowland, is the true value at 10° C. The gramme-degree is often called the lesser *calorie*, and the kilogramme-degree, which is 1000 times as large, is called the greater *calorie*. The following is a list of the principal units of heat which are in use.

$$1 \text{ joule} = 10^7 \text{ ergs.}$$

1 gramme degree or minor calorie	=	4·2 joules nearly.
1 kilogramme-degree or major calorie	=	4200 „ „
1 pound-degree Centigrade	=	1905 „ „
1 pound-degree Fahrenheit	=	1058 „ „

More precise Definition of the Secondary Unit.

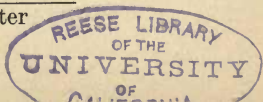
126. Rowland's results exhibit a difference of something like one per cent. between the specific heats of cold water at different temperatures. To render the definition of the secondary unit of heat exact, a standard temperature must be selected, the specific heat of water at this temperature must be called unity, and the secondary unit of heat will be n times the heat required to raise unit mass of water from this initial temperature through $1/n$ of a degree when n is indefinitely great.

127. When a secondary unit of heat is employed, the specific heat of water at some standard temperature is by definition unity; and the specific heat of any substance is a mere numerical ratio, namely, the ratio of the capacity of any quantity of that substance to the capacity of an equal mass of water (the water being at the standard temperature).

128. The *thermal capacity of unit volume* of a substance is another important element; we shall denote it by c . Let s denote the specific heat, and d the density of the substance; then c is the capacity of d units of mass, and therefore $c = sd$. The dimensions of c are the same as those of d , namely, $\frac{M}{L^3}$. Its numerical value will not be altered by any change in the units of length, mass, and time, which leaves the density of water unchanged.

In the C.G.S. system the value of c for water at ordinary temperatures is approximately unity; hence the value of c for any substance is approximately identical with the ratio

$$\frac{\text{heat given to the substance}}{\text{heat given to water}}$$



for the same rise of temperature, when the comparison is between equal volumes.

129. Mr. Herbert Tomlinson (Proc. Roy. Soc., June 19, 1885) has obtained the following determinations of specific heat from observations conducted in a uniform manner with metallic wires well annealed. The wires were heated sometimes to 60° C. and sometimes to 100° C., and were plunged in water at 20° . The formulæ are for the true specific heat at t° C. :—

Aluminium,.....	$\cdot 20700 + \cdot 000\ 2304\ t$
Copper,	$\cdot 09008 + \cdot 000\ 0648\ t$
German Silver,	$\cdot 09413 + \cdot 000\ 0106\ t$
Iron,	$\cdot 10601 + \cdot 000\ 140\ t$
Lead,	$\cdot 02998 + \cdot 000\ 031\ t$
Platinum,.....	$\cdot 03198 + \cdot 000\ 013\ t$
Platinum Silver,.....	$\cdot 04726 + \cdot 000\ 028\ t$
Silver,	$\cdot 05466 + \cdot 000\ 044\ t$
Tin,	$\cdot 05231 + \cdot 000\ 072\ t$
Zinc,	$\cdot 09009 + \cdot 000\ 075\ t$

The formulæ for the mean specific heat between 0° and t° are obtained from these by leaving the first term unchanged and halving the second term.

Violle has made the following determinations of specific heat at t° :—

Platinum,.....	$\cdot 0317 + 000\ 012\ t$
Iridium,.....	$\cdot 0317 + 000\ 012\ t$
Palladium,	$\cdot 0582 + 000\ 020\ t$

H. F. Weber has determined the specific heat of diamond to be

$$\cdot 0947 + \cdot 000\ 994\ t - \cdot 000\ 000\ 36\ t^2,$$

and consequently the mean specific heat of diamond from 0° to t° to be

$$\cdot 0947 + \cdot 000\ 497\ t - \cdot 000\ 000\ 12\ t^2.$$

The mean specific heat of ice according to Regnault is $\cdot 504$ between -20° and 0° , and $\cdot 474$ between -78° and 0°

130. The following list of specific heats of elementary substances is condensed from that given in Landolt and Börnstein's tables :—

Substance.	Temperature.	Sp. Heat.	Observer.
Aluminium,.....	15° to 97°	$\cdot 2122$	Regnault.
Antimony,.....	13 ,, 106	$\cdot 0486$	Bède.
Arsenic (crystalline),.....	21 ,, 68	$\cdot 0830$	{ Bettendorff & Wüllner.
,, (amorphous),.....	21 ,, 65	$\cdot 0758$	
Bismuth,.....	9 ,, 102	$\cdot 0298$	Bède.
Boron (crystalline),.....	0 ,, 100	$\cdot 2518$	Mixter & Dana
,, (amorphous),.....	18 ,, 48	$\cdot 254$	Kopp.
Bromine, solid,.....	-78 ,, -20	$\cdot 0843$	Regnault.
,, liquid,	13 ,, 45	$\cdot 1071$	Andrews.
Cadmium,	0 ,, 100	$\cdot 0548$	Bunsen.
Calcium,	0 ,, 100	$\cdot 1804$,,
Carbon, diamond,.....	11	$\cdot 1128$	H. F. Weber
,, graphite,.....	11	$\cdot 1604$,,
,, wood charcoal,..	0 to 99	$\cdot 1935$,,
Cobalt,	9 ,, 97	$\cdot 1067$	Regnault.
Copper,.....	15 ,, 100	$\cdot 0933$	Bède.
Gold,.....	0 ,, 100	$\cdot 0316$	Violle.
Iodine,.....	9 ,, 98	$\cdot 0541$	Regnault.
Iridium,.....	0 ,, 100	$\cdot 0323$	Violle.
Iron,	50	$\cdot 1124$	Byström.
Lead,	19 to 48	$\cdot 0315$	Kopp.
Lithium,	27 ,, 99	$\cdot 9408$	Regnault.
Magnesium,.....	20 ,, 51	$\cdot 245$	Kopp.
Manganese,	14 ,, 97	$\cdot 1217$	Regnault.
Mercury, solid,	-78 ,, -40	$\cdot 0319$,,
,, liquid,.....	17 ,, 48	$\cdot 0335$	Kopp.
Molybdenum,	5 ,, 15	$\cdot 0659$	{ De la Rive and Marcet.
Nickel,	14 ,, 97	$\cdot 1092$	
Osmium,	19 ,, 98	$\cdot 0311$,,

Substance.	Temperature.	Sp. Heat.	Observer.
Palladium,	0 to 100	·0592	Violle.
Phosphorus (yellow, solid) –	78 „ 10	·1699	Regnault.
„ („ liquid)	49 „ 98	·2045	Person.
„ (red),	15 „ 98	·1698	Regnault.
Platinum,	0 „ 100	·0323	Violle.
Potassium,	– 78 „ 0	·1655	Regnault.
Rhodium,	10 „ 97	·0580	Regnault.
Selenium, crystalline,	22 „ 62	·0840	{ Bettendorff & Wüllner.
Silicon, crystalline,	22	·1697	
Silver,	0 to 100	·0559	Bunsen.
Sodium,	– 28 „ 6	·2934	Regnault.
Sulphur (rhomb. cryst.),	17 „ 45	·163	Kopp.
„ (newly melted),	15 „ 97	·1844	Regnault.
Tellurium, crystalline,	21 „ 51	·0475	Kopp.
Thallium,	17 „ 100	·0335	Regnault.
Tin, cast,	0 „ 100	·0559	Bunsen.
Zinc,	0 „ 100	·0935	„

Substances not Elementary.

Brass (4 copper 1 tin), hard,	15° to 98°	·0858	Regnault.
„ „ soft,	14 „ 98	·0862	„
Ice,	– 20 „ 0	·504	„

131. The following determinations of specific heat of liquids are by Regnault. We have omitted decimal figures after the fourth, as even the second figure is different with different observers:—

	Alcohol.	Oil of Turpentine.		Ether.	Chloroform.	Bisulphide of Carbon.
– 20	·5053	·3842	– 30	·5113	·2293	·2303
0	·5475	·4106	0	·5290	·2324	·2352
40	·6479	·4538	30	·5468	·2354	·2401
80	·7694	·4842	60		·2384	
120		·5019				
160		·5068				

Schüller has found the specific heat of liquid benzine at t° to be

$$\cdot 37980 + \cdot 00144 t.$$

132. The following table (from Miller's Chemical Physics, p. 308) contains the results of Regnault's experiments on the specific heat of gases. The column headed "equal weights" contains the *specific heats* in the sense in which we have defined that term. The column headed "equal volumes" gives the relative thermal capacities of equal volumes at the same pressure and temperature:—

Thermal Capacities of Gases and Vapours at Constant Pressure.

Gas or Vapour.	Equal		Gas or Vapour.	Equal	
	Vols.	Weights.		Vols.	Weights.
Air, - -	·2375	·2375	Hydrochloric } Acid, - }	·2352	·1842
Oxygen, - -	·2405	·2175	Sulphuretted } Hydrogen, }	·2857	·2432
Nitrogen, -	·2368	·2438	Steam, - -	·2989	·4805
Hydrogen, -	·2359	3·4090	Alcohol, - -	·7171	·4534
Chlorine, -	·2964	·1210	Wood Spirit, -	·5063	·4580
Bromine, -	·3040	·0555	Ether, - -	1·2266	·4796
Nitrous Oxide, -	·3447	·2262	Ethyl Chloride	·6096	·2738
Nitric Oxide, -	·2406	·2317	„ Bromide	·7026	·1896
Carbonic Oxide	·2370	·2450	„ Disul- }	1·2466	·4008
Carbonic } Anhydride, }	·3307	·2163	„ phide, }	·8293	·4261
Carbonic Di- } sulphide, }	·4122	·1569	„ Cyanide,	·6461	·1566
Ammonia, -	·2996	·5084	Chloroform, -	·7911	·2293
Marsh Gas, -	·3277	·5929	Dutch Liquid,	1·2184	·4008
Olefiant Gas, -	·4106	·4040	Acetic Ether, -	1·0114	·3754
Arsenious } Chloride, }	·7013	·1122	Benzol, - -	·8341	·4125
Silicic Chloride	·7778	·1322	Acetone, -	2·3776	·5061
Titanic „	·8564	·1290	Oil of Tur- }	·6386	·1347
Stannic „	·8639	·0939	pentine, - }		
Sulphurous } Anhydride, }	·341	·154	Phosphorus } Chloride, - }		

133. E. Wiedemann (Pogg. Ann., 1876, No. 1, p. 39) has made the following determinations of the specific heats of gases :—

Specific Heat.

	At 0°.	At 100°.	At 200°.	Relative Density.
Air,	0·2389	1
Hydrogen,	3·410	0·0692
Carbonic Oxide,	0·2426	0·967
Carbonic Acid,	0·1952	0·2169	0·2387	1·529
Ethyl,	0·3364	0·4189	0·5015	0·9677
Nitrous Oxide,	0·1983	0·2212	0·2442	1·5241
Ammonia,	0·5009	0·5317	0·5629	0·5894

Multiplying the specific heat by the relative density, he obtains the following values of

Thermal Capacity of Equal Volumes.

	At 0°.	At 100°.	At 200°.
Air,	0·2389
Hydrogen,	0·2359
Carbonic Oxide, ..	0·2346
Carbonic Acid, ...	0·2985	0·3316	0·3650
Ethyl,	0·3254	0·4052	0·4851
Nitrous Oxide, ...	0·3014	0·3362	0·3712
Ammonia,	0·2952	0·3134	0·3318

The same author (Pogg. Ann., 1877, New Series, vol. ii. p. 195) has made the following determinations of specific heats of vapours at temperature t° :—

Vapour.	Range of Temp. in Experiments.	Specific Heat.
Chloroform,	26°·9 to 189°·8	·1341 + ·000 1354 t
Bromic Ethyl, ...	27°·9 to 189°·5	·1354 + ·000 3560 t
Benzine,	34°·1 to 179°·5	·2237 + ·001 0228 t
Acetone,	26°·2 to 179°·3	·2984 + ·000 7738 t
Acetic Ether, ...	32°·9 to 188°·8	·2738 + ·000 8700 t
Ether,	25°·4 to 188°·8	·3725 + ·000 8536 t

Regnault's determinations for the same vapours were as follows :—

Vapour.	Range of Temperature.	Mean Specific Heat for this Range.	
		According to Regnault.	According to Wiedemann.
Chloroform,.....	117 to 228	·1567	·1573
Bromic Ethel,...	77·7 to 196·5	·1896	·1841
Benzine,.....	116 to 218	·3754	·3946
Acetone,	129 to 233	·4125	·3946
Acetic Ether,....	115 to 219	·4008	·4190
Ether,	70 to 220	·4797	·4943

Regnault has also determined the mean specific heat of bisulphide of carbon vapour between 80° and 147° to be ·1534, and between 80° and 229° to be ·1613.

MELTING POINTS AND HEATS OF LIQUEFACTION.

134. The following table of melting points and latent heats of liquefaction is taken from the German edition of this work. Those which are marked with an asterisk are recent very careful determinations by Violle.

	Melting Point.	Latent Heat.		Melting Point.	Latent Heat.
Iridium,	1950*		Cadmium,	315	13·6
Platinum,	1775*	27·2*	Bismuth,	260	12·6
Palladium,	1500*	36·3*	Tin,	230	14·6
Iron,	1600		Selenium,	217	
Nickel,	1450	4·64	Sulphur,	115	9·4
Cast steel,	1370		Sodium,	96	
Pig iron,	1075		Potassium,	62	
Copper,	1054*		Phosphorus,	44	5·0
Gold,	1045*		Paraffin,	54	
Silver,	954*	24·7	Bromine,	-7	
Glass,	1100		Mercury,	-40	2·82
Aluminium,	600		Iodine,		11·7
Antimony,	432		Nitrate of soda,		63·0
Zinc,	412	28·1	Nitrate of potash,		47·4
Lead,	326	5·4	Ice (Bunsen),		80

Violle states that 1950° , the melting point of iridium, is very near to that of the hottest part of the oxy-hydrogen flame.

The latent heat of liquefaction of ice was found by Regnault, and by Provostaye and Desains, to be 79° . Bunsen (Pogg. Ann. cxli., p. 30) obtained the value 80.025. He found the specific gravity of ice to be .9167.

135. Chandler Roberts and Wrightson have compared the densities of molten and solid metals by weighing a solid metal ball in a bath of molten metal either of the same or a different kind (Phys. Soc., 1881, p. 195, and 1882, p. 102). They find that "iron expands rapidly (as much as 6 per cent.) in cooling from the liquid to the plastic state, and then contracts 7 per cent. to solidity; whereas bismuth appears to expand in cooling from the liquid to the solid state about 2.35 per cent." The following is their tabular statement of results:—

Metal.	Sp. Grav. of Solid.	Sp. Grav. of Liquid.	Percentage of change in volume from cold solid to liquid.
Bismuth,	9.82	10.055	Decrease of vol. 2.3
Copper,	8.8	8.217	Increase of vol. 7.1
Lead,	11.4	10.37	„ 9.93
Tin,	7.5	7.025	„ 6.76
Zinc,	7.2	6.48	„ 11.1
Silver,	10.57	9.51	„ 11.2
Iron (No. 4 foundry, Cleveland), }	6.95	6.88	„ 1.02

136. Change of volume in melting, from Kopp's experiments (Watts' Dict., Art. *Heat*, p. 78):—

Phosphorus. Calling the volume at 0° unity, the volume at the melting point (44°) is 1.017 in the solid, and 1.052 in the liquid, state.

Sulphur. Volume at 0° being 1, volume at the melting point (115°) is 1.096 in the solid, and 1.150 in the liquid, state.

Wax. Volume at 0° being 1, volume at melting point (64°) is 1·161 in solid, and 1·166 in liquid, state.

Stearic Acid. Volume at 0° being 1, volume at melting point (70°) is 1·079 in solid, and 1·198 in liquid, state.

Rose's Fusible Metal (2 parts bismuth, 1 tin, 1 lead). Volume at 0° being 1, volume at 59° is a maximum, and is 1·0027. Volume at melting point (between 95° and 98°) is greater in liquid than in solid state by 1·55 per cent.

137. The following table (from Miller's Chemical Physics, p. 344) exhibits the change of volume of certain substances in passing from the liquid to the vaporous condition under the pressure of one atmosphere :—

1 volume of water yields	1696 volumes of vapour.
„ alcohol	528 „ „
„ ether	298 „ „
„ oil of turpentine	193 „ „

138. The following table of boiling points and heats of vaporization, at atmospheric pressure, is condensed from Landolt and Börnstein, pp. 189, 190 :—

	Boiling Point.	Latent Heat of Vaporization.	Observer.
Alcohol,	77·9	202·4	Andrews.
Bisulphide of Carbon,	46·2	86·7	„
Bromine,	58	45·6	„
Ether,	34·9	90·4	„
Mercury,	350	62·0	Person.
Oil of Turpentine,	159·3	74·0	Brix.
Water,	100	535·9	Andrews.

A specially careful determination of the boiling point of sulphur has been made by Callendar and Griffiths (Proc. R. S., Dec. 1890, p. 58). The result is, “the temperature, by normal air thermometer at constant pressure, of the saturated vapour of sulphur boiling freely under a pressure of 760 mm. of mercury at 0° C. and $g = 980·61$ C.G.S. (sea level in lat. 45°), is 444°·53 C.”

139. Regnault's approximate formula for what he calls "the total heat of steam at t° ," that is, for the heat required to raise unit mass of water from 0° to t° in the liquid state and then convert it into steam at t° , is

$$606\cdot5 + \cdot305 t.$$

If the specific heat of water were the same at all temperatures, this would give

$$606\cdot5 - \cdot695 t$$

as the heat of evaporation at t° . But since, according to Regnault, the heat required to raise the water from 0° to t° is

$$t + \cdot000\ 02\ t^2 + \cdot000\ 000\ 3\ t^3,$$

the heat of evaporation will be the difference between this and the "total heat," that is, will be

$$606\cdot5 - \cdot695 t - \cdot000\ 02\ t^2 - \cdot000\ 000\ 3\ t^3,$$

which is accordingly the value adopted by Regnault as the heat of evaporation of water at t° .

140. According to Regnault, the increase of pressure at constant volume, and increase of volume at constant pressure, when the temperature increases from 0° to 100° , have the following values for the gases named :—

Gas.	At Constant Volume.	At Constant Pressure.
Hydrogen,	$\cdot3667$	$\cdot3661$
Air,	$\cdot3665$	$\cdot3670$
Nitrogen,	$\cdot3668$...
Carbonic Oxide,	$\cdot3667$	$\cdot3669$
Carbonic Acid,	$\cdot3688$	$\cdot3710$
Nitrous Oxide,	$\cdot3676$	$\cdot3719$
Sulphurous Acid,	$\cdot3845$	$\cdot3903$
Cyanogen,	$\cdot3829$	$\cdot3877$

Jolly has obtained the following values for the coefficient of increase of pressure at constant volume :—

Air,	·00366957
Oxygen,.....	·00367430
Hydrogen,.....	·00365620
Nitrogen,	·0036677
Carbonic Acid,.....	·0037060
Nitrous Oxide,.....	·0037067

Rowland, from his own experiments, has deduced for air at constant volume, the value ·0036675 or ·0036707, according as Regnault's or Wüllner's determination of the expansion of mercury is employed in the calculation.

141. According to recent researches by P. Chappuis, the mean coefficient of increase of pressure of hydrogen at constant volume from 0° to 100° (the pressure at 0° being that of 100 centims. of mercury) is ·00366254; and taking this as the standard of "uniform" increase the true coefficient of increase of pressure for nitrogen, with the same pressure at 0° , is

$$\cdot 0036770 - 7 \cdot 8267 \times 10^{-8} t + 4 \cdot 780 \times 10^{-10} t^2;$$

and for carbonic acid,

$$\cdot 00373537 - 2 \cdot 6754 \times 10^{-7} t + 2 \cdot 6157 \times 10^{-10} t^2 \\ + 7 \cdot 5992 \times 10^{-12} t^3.$$

142. *Pressure of Aqueous Vapour.*

Temp.	Cm. Mercury in Lat. 45° at Sea Level.	Dynes per sq. cm.	Temp.	Cm. Mercury in Lat. 45° at Sea Level.	Dynes per sq. cm.
99·0	73·316	$9 \cdot 775 \times 10^5$	100·0	76·000	$1 \cdot 0133 \times 10^6$
·1	·581	9·810	·1	·273	1·0169
·2	·846	9·846	·2	·547	1·0206
·3	74·113	9·881	·3	·821	1·0242
·4	·380	9·917	·4	77·097	1·0279
·5	·648	9·953	·5	·373	1·0316
·6	·917	9·988	·6	·650	1·0353
·7	75·186	$1 \cdot 0024 \times 10^6$	·7	·928	1·0390
·8	·457	1·0060	·8	78·207	1·0427
·9	·728	1·0097	·9	·486	1·0464
100·0	76·000	1·0133	101·0	·767	1·0502

The above table, showing the maximum pressure at temperatures near 100° C., is based on Regnault's determinations as revised by Broch.

143. Maximum pressure of aqueous vapour at various temperatures, in dynes per sq. centim.

Temp.	Pressure.	Temp.	Pressure.	Temp.	Pressure.
-20°	1.26×10^3	20°	2.315×10^4	80°	4.731×10^5
-15	1.92 „	25	3.135 „	90	7.006 „
-10	2.87 „	30	4.201 „	100	1.0133×10^6
-5	4.21 „	40	7.315 „	120	1.989 „
0	6.09 „	50	1.226×10^5	140	3.624 „
5	8.67 „	60	1.985 „	160	6.204 „
10	1.219×10^4	70	3.111 „	200	1.559×10^7
15	1.690 „				

144. The following are approximate values of the maximum pressure of aqueous vapour at various temperatures, in millimetres of mercury. They can be reduced to dynes per sq. cm. by multiplying by 1334:—

	mm.		mm.		mm.		mm.
0°	4.6	92°	567	112°	1150	132°	2155
10	9.2	94	611	114	1228	134	2286
20	17.4	96	658	116	1311	136	2423
30	31.5	98	707	118	1399	138	2567
40	54.9	100	760	120	1491	140	2718
50	92.0	102	816	122	1588	142	2875
60	149	104	875	124	1691	144	3040
70	233	106	938	126	1798	146	3213
80	355	108	1004	128	1911	148	3393
90	525	110	1075	130	2030	150	3581

145. The density (in gm. per cub. cm.) of aqueous vapour at any temperature t and any pressure p (dynes per sq. cm.), whether equal to or less than the maximum pressure, is

$$\frac{.622 \times .001276}{1 + .00366 t} \times \frac{p}{10^6}.$$

If q denote the pressure in millimetres of mercury, the approximate formula is

$$\frac{.622 \times .001293}{1 + .00366 t} \times \frac{q}{760}.$$

146. Temperature of evaporation and dew-point (Glaisher's Tables, second edition, page iv.). The following are the factors by which it is necessary to multiply the excess of the reading of the dry thermometer over that of the wet, to give the excess of the temperature of the air above that of the dew-point :—

Reading of Dry Bulb Therm.	Factor.	Reading of Dry Bulb Therm.	Factor.
- 10° C. = 14° F.	8.76	15° C. = 59° F.	1.89
- 5 23	7.28	20 68	1.79
0 32	3.32	25 77	1.70
+ 5 41	2.26	30 86	1.65
+ 10 50	2.06	35 95	1.60

For density of moist air see end of this chapter.

147. *Maximum Pressure of various Vapours, in dynes per sq. cm.*

	Alcohol.	Ether.	Sulphide of Carbon.	Chloroform.
- 20°	4455	9.19×10^4	6.31×10^4	
- 10	8630	1.53×10^5	1.058×10^5	
0	16940	2.46 „	1.706 „	
10	32320	3.826 „	2.648 „	
20	59310	5.772 „	3.975 „	2.141×10^5
30	1.048×10^5	8.468 „	5.799 „	3.301 „
40	1.783 „	1.210×10^6	8.240 „	4.927 „
50	2.932 „	1.687 „	1.144×10^6	7.14 „
60	4.671 „	2.301 „	1.554 „	1.007×10^6
80	1.084×10^6	4.031 „	2.711 „	1.878 „
100	2.265 „	6.608 „	4.435 „	3.24 „
120	4.31 „	1.029×10^7	6.87 „	5.24 „

Critical Temperatures and Pressures.

148. The following table of critical temperatures of gases (above which they cannot be liquefied), and the corresponding maximum pressures, is given by van der Waals (p. 464 of English translation, *Phys. Soc., Lond.*):—

	Critical.		
	Temp. °	Pressure. atm.	
Carbonic Acid,	30·92	73	Andrews.
Ether,	190	36·9	Sajotschewsky.
Carbon Bisulphide,	271·8	74·7	„
„ „	273	77·9	Hannay.
Sulphurous Acid,	155·4	78·9	Sajotschewsky.
Alcohol,	234·3	62·1	„
„	234·6	65	Hannay.
„	256	119	Cagniard.
Monochlorethane,	182·5	52·6	Sajotschewsky.
Benzol,	280·6	49·5	„
Acetone,	232·8	52·2	„
„	237·5	60	„
Chloroform,	260	54·9	„
Ethylene,	9·2	58	van der Waals.
Hydrochloric Acid,	51·25	86	Ansdell.
Acetylene,	37	68	„
Carbon Tetrachloride,	277·9	58·1	Hannay.
Nitrous Oxide,	36·4	73·1	Janssen.

Conductivity.

149. By the *thermal conductivity* of a substance at a *given temperature* is meant the value of k in the expression

$$Q = k A \frac{v_2 - v_1}{x} t, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where Q denotes the quantity of heat that flows, in time t , through a plate of the substance of thickness x , the area of each of the two opposite faces of the plate being A ,

and the temperatures of these faces being respectively v_1 and v_2 , each differing but little from the given temperature. The lines of flow of heat are supposed to be normal to the faces, or, in other words, the isothermal surfaces within the plate are supposed to be parallel to the faces; and the flow of heat is supposed to be *steady*, in other words, no part of the plate is to be gaining or losing heat on the whole.

Briefly, and subject to these understandings, conductivity may be defined as *the quantity of heat that passes in unit time, through unit area of a plate whose thickness is unity, when its opposite faces differ in temperature by one degree.*

150. *Dimensions of Conductivity.* From equation (1) we have

$$k = \frac{Q}{v_2 - v_1} \cdot \frac{x}{At} \cdot \cdot \cdot \cdot \cdot \quad (2)$$

The dimensions of Q and $v_2 - v_1$ are respectively $M\Delta$ and Δ , giving M as the dimensions of the first factor. The

dimensions of the factor $\frac{x}{At}$ are $\frac{L}{LT}$; hence the dimensions of k are $\frac{M}{LT}$. This is on the supposition that the

unit of heat is the heat required to raise *unit mass* of water one degree. In calculations relating to conductivity it is perhaps more usual to adopt as the unit of heat the heat required to raise *unit volume* of water one

degree. The dimensions of $\frac{Q}{v_2 - v_1}$ will then be L^3 , and

the dimensions of k will be $\frac{L^2}{T}$. In the C.G.S. system

these two units of heat are practically identical.

151. Let c denote the thermal capacity of unit volume of a substance through which heat is being conducted. Then $\frac{k}{c}$ denotes a quantity whose value it is often necessary to discuss in investigations relating to the transmission of heat. We have, from equation (2),

$$\frac{k}{c} = \frac{Q'}{v_2 - v_1} \cdot \frac{x}{At},$$

where Q' denotes $\frac{Q}{c}$. Hence $\frac{k}{c}$ would be the numerical value of the conductivity of the substance, if the unit of heat employed were the heat required to raise *unit volume of the substance* one degree. Professor Clerk Maxwell proposed to call $\frac{k}{c}$ the *thermometric* conductivity, as distinguished from k the *thermal* or *calorimetric* conductivity. We prefer, in accordance with Sir Wm. Thomson's article, "Heat," in the Encyclopædia Britannica to call $\frac{k}{c}$ the *diffusivity* of the substance for heat, a name which is based on the analogy of $\frac{k}{c}$ to a coefficient of diffusion.

Coefficient of Diffusion.

152. By the coefficient of diffusion of a given substance in a given solution is meant the value of K in the expression

$$m = K A \frac{s_2 - s_1}{x} t, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where m denotes the mass of the substance that passes in time t through a layer of the solution of thickness x , the area of each face of the layer being A , and the con-

centrations at these faces s_1 and s_2 ; concentration being so defined that in a solution of uniform concentration the mass of the substance is equal to the volume of the solution multiplied by the concentration.

Since m divided by $s_2 - s_1$ is a mass divided by a concentration, and is therefore a volume, the dimensions of K are found to be $\frac{L^2}{T}$.

Let y denote the thickness of a layer of concentration s_1 , which would be raised to concentration s_2 by the mass of the substance transmitted in time t . The mass required is $(s_2 - s_1) y A$, which when put for m in equation (3) gives

$$K = \frac{yx}{t} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot (4)$$

In the analogous problem for heat, denoting by y the thickness of a stratum at temperature v_1 , which would be raised to v_2 by the heat transmitted in time t , we must put $c(v_2 - v_1) y A$ for Q in equation (1) or (2), and we deduce

$$\frac{k}{c} = \frac{yx}{t} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot (5)$$

Hence $\frac{k}{c}$ is the analogue of K .

153. The following table gives the coefficients of diffusion between certain inorganic liquids and water according to Scheffer's observations. His units are the centimetre and the day; hence to reduce these coefficients to C.G.S. values they must be divided by the number of seconds in a day, that is by 86400. The first column gives the concentration in grammes per cub. cm., and the other columns the values of m in grammes.

Substance in 100 cub. cm. of solution.	3½° C. HCl.	5½° C. NaCl.	2½° C. NaNO ₃ .	7½° C. AgNO ₃ .	10½° C. Na ₂ S ₂ O ₃ .
4·55 gm.	1·622
4·96	·899	...
5·45	...	·756
5·6	·635
10·35	·622
12·53	...	·727
22·7	2·01
26·3	...	·732
26·88	·543
35·97	·774	...
49·09	·565
65·58	·649	...

Temperature-Coefficient.

De Heen finds that the coefficients of diffusion of solutions of the undermentioned salts are affected with the following factors depending on the temperature t :—

MgSO ₄	1 - ·000 119 t	K ₂ CO ₃	1 - ·000 127 t
KNO ₃	1 - ·000 127 t	Na ₂ HPO ₄	1 - ·000 128 t

154. The following values of K in terms of the centimetre and second are given in Professor Clerk Maxwell's Theory of Heat, 4th edition, p. 332, on the authority of Professor Loschmidt of Vienna.

Coefficients of Interdiffusion of Gases.

Carbonic Acid and Air,.....	·1423
„ „ Hydrogen,.....	·5614
„ „ Oxygen,.....	·1409
„ „ Marsh Gas,.....	·1586
„ „ Carbonic Oxide,.....	·1406
„ „ Nitrous Oxide,	·0982
Oxygen and Hydrogen,	·7214
„ „ Carbonic Oxide,.....	·1802
Carbonic Oxide and Hydrogen,.....	·6422
Sulphurous Acid and Hydrogen,.....	·4800

These may be compared with the value of $\frac{k}{c}$ for air, which, according to Professor J. Stefan of Vienna, is $\cdot 256$.

The value of k for air, according to the same authority, is $5\cdot 58 \times 10^{-5}$, and is independent of the pressure. Professor Maxwell, by a different method, calculates its value at $5\cdot 4 \times 10^{-5}$.

Results of Experiments on Conductivity of Solids.

155. Principal Forbes' results for the conductivity of iron (Stewart on Heat, p. 261, second edition) are expressed in terms of the foot and minute, the cubic foot of water being the unit of thermal capacity. Hence the value of Forbes' unit of conductivity, when referred to C.G.S., is $\frac{(30\cdot 48)^2}{60}$, or $15\cdot 48$; and his results must be multiplied by $15\cdot 48$ to reduce them to the C.G.S. scale. His observations were made on two square bars; the side of the one being $1\frac{1}{4}$ inch, and of the other an inch. The results when reduced to C.G.S. units are as follows:—

Temp. Cent.		$1\frac{1}{4}$ -inch bar.		1-inch bar.
0°	·207	·1536
25	·1912	·1460
50	·1771	·1399
75	·1656	·1339
100	·1567	·1293
125	·1496	·1259
150	·1446	·1231
175	·1399	·1206
200	·1356	·1183
225	·1317	·1160
250	·1279	·1140
275	·1240	·1121

156. Neumann's results (Ann. de Chim., vol. lxvi. p.

185) must be multiplied by $\cdot 000848$ to reduce them to our scale. They then become as follows:—

Copper,.....	1·108
Brass,.....	·302
Zinc,	·307
Iron,	·164
German Silver,	·109
Ice,.....	·0057

In the same paper he gives for the following substances the values of the diffusivity $\frac{k}{c}$. These require the same reducing factor as the values of k , and when reduced to our scale are as follows:—

	Values of $\frac{k}{c}$
Coal,.....	·00116
Melted Sulphur,	·00142
Ice,.....	·0114
Snow,.....	·00356
Frozen Mould,.....	·00916
Sandstone,.....	·0136
Granite (coarse),	·0109
Serpentine,	·00594

157. Sir W. Thomson's results, deduced from observations of underground thermometers at three stations at Edinburgh (Trans. R. S. E., 1860, p. 426), are given in terms of the foot and second, the thermal capacity of a cubic foot of water being unity, and must be multiplied by $(30\cdot48)^2$ or 929 to reduce them to our scale. The following are the reduced results:—

	k , or Conductivity.	$\frac{k}{c}$
Trap-rock of Calton Hill,.....	·00415	·00786
Sand of experimental garden,	·00262	·00872
Sandstone of Craigleith Quarry,.....	·01068	·02311

The values of $\frac{k}{c}$ were directly derived from the observations; and the values of k were deduced from them by the help of determinations of c made by Regnault.

My own result for the value of $\frac{k}{c}$ from the Greenwich underground thermometers (Greenwich Observations, 1860) is in terms of the French foot and the year. As a French foot is 32.5 centims., and a year is 31557000 seconds, the reducing factor is $(32.5)^2 \div 31557000$; that is, 3.347×10^{-5} . The result is

$\frac{k}{c}$

Gravel of Greenwich Observatory Hill,..... .01249

Professors Ayrton and Perry (Phil. Mag., April, 1878) determined the conductivity of a Japanese building stone (porphyritic trachyte) to be .0059.

158. Ångström, in Pogg. Ann., vols. cxiv. (1861) and cxviii. (1863), employs as units the centimetre and the minute; hence his results must be divided by 60. These results, as given at p. 429 of his second paper, will then stand as follows:—

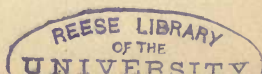
	Value of $\frac{k}{c}$
Copper, first specimen,.....	1.216 (1 - .00214 <i>t</i>)
„ second specimen,.....	1.163 (1 - .001519 <i>t</i>)
Iron,224 (1 - .002874 <i>t</i>)

He adopts for c the values

.84476 for copper, .88620 for iron,

and thus deduces the following values of k :—

	Conductivity.
Copper, first specimen,.....	1.027 (1 - .00214 <i>t</i>)
„ second specimen,.....	.983 (1 - .001519 <i>t</i>)
Iron,199 (1 - .002874 <i>t</i>)



159. A Committee, consisting of Professors Herschel and Lebour, and Mr. J. F. Dunn, appointed by the British Association to determine the thermal conductivities of certain rocks, have obtained results from which the following selection has been made by Professor Herschel :—

Substance.	Conductivity in C.G.S. Units.	$\frac{k}{c}$
Iron pyrites, more than.....	·01 more than	·0170
Rock Salt, rough crystal,.....	·0113	·0288
Fluorspar, rough crystal,.....	·00963	·0156
Quartz, opaque crystal, and quartzites,.....	·0080 to ·0092	·0175 to ·0190
Silicious sandstones (slightly wet),	·00641 to ·00854	·0130 to ·0230
Galena, rough crystal, inter- spersed with quartz,.....	·00705	·0171
Sandstone and hard grit, dry,...	·00545 to ·00565	·0120
Sandstone and hard grit, thor- oughly wet,	·00590 to ·00610	·0100
Micaceous flagstone, along the cleavage,.....	·00632	·0116
Micaceous flagstone, across cleavage,.....	·00441	·0087
Slate, along cleavage,	·00550 to ·00650	·0102
Do., across cleavage,.....	·00315 to ·00360	·0057
Granite, various specimens, about	·00510 to ·00550	·0100 to ·0120
Marbles, limestone, calcite, and compact dolomite,	·00470 to ·00560	·0085 to ·0095
Red Serpentine (Cornwall),	·00441	·0065
Caen stone (building limestone),	·00433	·0089
Whinstone, trap rock, and mica schist,	·00280 to ·00480	·0055 to ·0095
Clay slate (Devonshire),	·0272	·00053
Tough clay (sun-dried),	·00223	·0048
Do., soft (with one-fourth of its weight of water),	·00310	·0035
Chalk,	·00200 to ·00330	·0046 to ·0059

Substance.	Conductivity in C.G.S. Units.	$\frac{k}{c}$.
Calcareous sandstone (firestone),	·00211	·0049
Plate-glass, German and English,	·00198 to ·00234	·00395 to ?
German glass toughened,.....	·00185	·00395
Heavyspar, opaque rough crystal,	·00177
Fire-brick,	·00174	·0053
Fine red brick,	·00147	·0044
Fine plaster of Paris, dry plate,	·00120	·0060
Do., thoroughly wet,	·00160	·0025
White sand, dry,	·00093	·0032
Do., saturated with water, about.....	·00700	·0120 about
House coal and cannel coal,	·00057 to ·00113	·0012 to ·0027
Pumice stone,	·00055

160. Pécelet in *Annales de Chimie*, sér. 4, tom. ii. p. 114 [1841], employs as the unit of conductivity the transmission, in one second, through a plate a metre square and a millimetre thick, of as much heat as will raise a cubic decimetre (strictly a kilogramme) of water one degree. Formula (2), § 150, shows that the value of this conductivity in the C.G.S. system, is

$$\frac{1000}{1} \cdot \frac{1}{10000}; \text{ that is, } \frac{1}{100}.$$

His results must accordingly be divided by 100.

The same author published in 1853 a greatly extended series of observations, in a work entitled "*Nouveaux documents relatifs au chauffage et à la ventilation.*" In this series the conductivity which is adopted as unity is the transmission, in one hour, through a plate a metre square and a metre thick, of as much heat as will raise a kilogramme of water one degree. This conductivity in C.G.S. units is

$$\frac{1000}{1} \cdot \frac{100}{10000} \cdot \frac{1}{3600}; \text{ that is, } \frac{1}{360}.$$

The results must therefore be divided by 360. Those of them which refer to metals appear to be much too small. The following are for badly conducting substances :—

	Density.	Conductivity.
Fir, across fibres,.....	·48	·00026
„ along fibres,	·48	·00047
Walnut, across fibres,.....	...	·00029
„ along fibres,	·00048
Oak, across fibres,	·00059
Cork,.....	·22	·00039
Caoutchouc,.....	...	·00041
Gutta Percha,	·00048
Starch Paste,	1·017	·00118
Glass,	2·44	·0021
Glass,	2·55	·0024
Sand, quartz,	1·47	·00075
Brick, pounded, coarse-grained,	1·0	·00039
„ passed through } silk sieve,..... }	1·16	·00046
Fine brick dust, obtained by de- } cantation,..... }	1·55	·00039
Chalk, powdered, slightly damp,	·92	·00030
„ washed and dried,	·85	·00024
„ washed, dried, and } compressed,..... }	1·02	·00029
Potato-starch,	·71	·00027
Wood-ashes,	·45	·00018
Mahogany sawdust,	·31	·00018
Wood charcoal, ordinary, powdered,	·49	·00022
Bakers' breeze, in powder, passed } through silk sieve,	·25	·00019
Ordinary wood charcoal in pow- } der, passed through silk sieve, }	·41	·000225
Coke, powdered,	·77	·00044
Iron filings,	2·05	·00044
Binoxide of Manganese,.....	1·46	·00045

Woolly Substances.

Cotton Wool, of all densities,	·000111
Cotton swansdown (molleton de } coton), of all densities,	·000111
Calico, new, of all densities,.....	...	·000139
Wool, carded, of all densities,	·000122

	Density.	Conductivity.
Woollen swansdown (molleton de laine) of all densities),.....}	...	·000067
Eider-down,	·000108
Hempen cloth, new,.....	·54	·000144
„ old,	·58	·000119
Writing-paper, white,	·85	·000119
Grey paper, unsized,	·48	·000094

161. In Professor George Forbes's paper on conductivity (Proc. R.S.E., February, 1873) the units are the centim. and the minute; hence his results must be divided by 60.

In a letter dated March 4, 1884, to the author of this work, Professor Forbes remarks that the mean temperature of the substances in these experiments was -10° , and expresses the opinion that bad conductors (such as most of these substances) conduct worse at low than at high temperatures—an opinion which was suggested by the analogy of electrical insulators. His results reduced to C.G.S. are—

Ice, along axis,	·00223	Kamptulikon,.....	·00011
Ice, perpendicular to axis,	·00213	Vulcanized india-rubber,	·000 089
Black marble,.....	·00177	Horn,	·000 087
White marble,.....	·00115	Beeswax,.....	·000 087
Slate,.....	·00081	Felt,.....	·000 087
Snow,	·00072	Vulcanite,	·000 0833
Cork,	·000717	Haircloth,	·000 0402
Glass,.....	·0005	Cotton-wool, divided,	·000 0433
Pasteboard,.....	·000453	„ pressed,	·000 0335
Carbon,	·000405	Flannel,	·000 0355
Roofing-felt,.....	·000335	Coarse linen,.....	·000 0298
Fir, parallel to fibre,	·0003	Quartz, along axis,	·000922
Fir, across fibre, and along radius,.....}	·000088	„ „	·00124
Boiler-cement,.....	·000162	„ „	·00057
Paraffin,	·00014	„ „	·00083
Sand, very fine,.....	·000131	Quartz, perpendicular to axis,.....}	·0040
Sawdust,	·000123	„ „	·0044

Professor Forbes quotes a paper by M. Lucien De la

Rive (Soc. de Ph. et d'Hist. Nat. de Genève, 1864) in which the following result is obtained for ice,

Ice,..... ·00230

M. De la Rive's experiments are described in *Annales de Chimie*, sér. 4, tom. i. pp. 504-6.

Conductivity of Liquids.

162. H. F. Weber (Sitz. kön. Preuss. Akad., 1885), has made the following determinations of conductivities of liquids at temperatures of from 9° to 15° C. He employs the centimetre, the gramme, and the minute as units : we have accordingly divided the original numbers by 60 to reduce to C.G.S.

	Conduc- tivity.		Conduc- tivity.
Water,.....	·00136	Amyl Acetate,.....	·000302
Aniline,.....	·000408	Chloro Benzol,.....	·000302
Glycerine,	·000670	Chloroform,.....	·000288
Ether,.....	·000303	Chloro Carbon,	·000252
Methyl Alcohol,	·000495	Propyl Chloride,	·000283
Ethyl Alcohol,	·000423	Isobutyl Chloride,.....	·000278
Propyl Alcohol,	·000373	Amyl Chloride,.....	·000284
Butyl Alcohol,.....	·000340	Bromo Benzol,.....	·000265
Amyl Alcohol,.....	·000328	Ethyl Bromide,.....	·000247
Formic Acid,.....	·000648	Propyl Bromide,	·000257
Acetic Acid,.....	·000472	Isobutyl Bromide,.....	·000278
Propion Acid,.....	·000390	Amyl Bromide,.....	·000237
Butyric Acid,.....	·000360	Ethyl Iodide,.....	·000222
Isobutyric Acid,	·000340	Propyl Iodide,.....	·000220
Valerian Acid,.....	·000325	Isobutyl Iodide,.....	·000208
Isovalerian Acid,.....	·000312	Amyl Iodide,.... .	·000203
Isocapron Acid,.....	·000298		
Methyl Acetate,	·000385	Benzol,.....	·000333
Ethyl Formiate,.....	·000378	Toluol,.....	·000307
Ethyl Acetate,.....	·000348	Cymol,.....	·000272
Propyl Formiate,	·000357	Oil of Turpentine,.....	·000260
Propyl Acetate,	·000327	Sulphuric Acid,.....	·000765
Methyl Butyrate,.....	·000335	Bisulphide of Carbon,	·000343
Ethyl Butyrate,	·000318	Oil of Mustard,.....	·000382
Methyl Valerate,	·000315	Ethyl Sulphide,.....	·000328
Ethyl Valerate,.....	·000307		

163. C. Chree (Proc. Roy. Soc., April, 1887) found, by two slightly different methods, at temperatures between 18° and 20° , values which, when divided by 60 to reduce them to C.G.S., are :—

	Conductivity.	
	1st Method.	2nd Method.
Water,	·001245	·001358
Methylated Spirit,	·000590	·000577
Paraffin Oil,	·000440	·000455
Turpentine Oil,	·000315
Bisulphide of Carbon, ...	·000537	...

More recently, Grätz has found at 13° :—

	Conductivity.		Conductivity.
Glycerine,	·000637	Ethyl Alcohol,	·000545
Ether,	·000378	Bisulphide of Carbon,	·000266
Oil of Turpentine, ...	·000325	Chloride of Sodium Solution, density 1·152,	·000112

Assuming $k_t = k_0(1 - \alpha t)$, where k_0 , k_t denote the conductivities at 0° and t° , the temperature-coefficient α had, in the neighbourhood of 13° , the values :—

Glycerine,	·012	Oil of Turpentine,	·007
Sodium Chloride Solution,	·006		

Conductivity of Gases.

164. Winkelmann gives the following values for gases at $7^{\circ}5$:—

	Conductivity.		Conductivity.
Air,	·0000516	Carbonic Acid, ...	·0000273
Hydrogen,	·00033	Ethyl,	·0000356
Nitrous Oxide, ...	·0000312	Marsh Gas,	·000065

For the temperature-coefficient (defined as in the preceding section) he finds :—

Air,	·000208	Carbonic Acid,	·0038
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Emission and Surface Conduction.

165. Mr. D. M'Farlane has published (Proc. Roy. Soc., 1872, p. 93) the results of experiments on the loss

of heat from blackened and polished copper balls in air at atmospheric pressure. They need no reduction, the units employed being the centimetre, gramme, and second. The general result is expressed by the formulæ

$$x = \cdot 000238 + 3\cdot 06 \times 10^{-6} t - 2\cdot 6 \times 10^{-8} t^2$$

for a blackened surface, and

$$x = \cdot 000168 + 1\cdot 98 \times 10^{-6} t - 1\cdot 7 \times 10^{-8} t^2.$$

for polished copper, x denoting *emissivity*, that is, the quantity of heat lost per second per square centim. of surface of the copper, per degree of difference between its temperature and that of the walls of the enclosure. These latter were blackened internally, and were kept at a nearly constant temperature of 14°C . The air within the enclosure was kept moist by a saucer of water. The greatest difference of temperature employed in the experiments (in other words, the highest value of t) was 50° or 60°C .

The following table contains the values of x calculated from the above formulæ, for every fifth degree, within the limits of the experiments :—

Difference of Temperature.	Emissivity.		Ratio.
	Polished Surface.	Blackened Surface.	
5	$\cdot 000178$	$\cdot 000252$	$\cdot 707$
10	$\cdot 000186$	$\cdot 000266$	$\cdot 699$
15	$\cdot 000193$	$\cdot 000279$	$\cdot 692$
20	$\cdot 000201$	$\cdot 000289$	$\cdot 695$
25	$\cdot 000207$	$\cdot 000298$	$\cdot 694$
30	$\cdot 000212$	$\cdot 000306$	$\cdot 693$
35	$\cdot 000217$	$\cdot 000313$	$\cdot 693$
40	$\cdot 000220$	$\cdot 000319$	$\cdot 693$
45	$\cdot 000223$	$\cdot 000323$	$\cdot 690$
50	$\cdot 000225$	$\cdot 000326$	$\cdot 690$
55	$\cdot 000226$	$\cdot 000323$	$\cdot 690$
60	$\cdot 000226$	$\cdot 000328$	$\cdot 690$

166. Rowland (p. 170 of paper on Mechanical Equivalent) found for the coefficient of total emission of heat from a nickel-plated calorimeter to the surrounding jacket, the jacket being at 20° :—

Difference of Temp.	Coefficient of Emission.
0	·000081
5	·000082
10	·000086
15	·000089
20	·000093
25	·000096

and points out that these are rather less than half the values obtained by M'Farlane for polished copper. The jacket was not blackened.

Influence of Size.

According to Prof. Ayrton, who quotes a table in "Box on Heat," the coefficient of emission increases as the size of the emitting body diminishes, and for a blackened sphere of radius r centims. may be stated as

$$·000\ 4928 + \frac{·000\ 3609}{r}.$$

The value of r in M'Farlane's experiments was 2.

Radiation in Vacuo.

167. Dr. J. T. Bottomley (Phil. Trans., 1888, and subsequently in 1890, the final results being not hitherto published), experimenting with the same two globes employed by M'Farlane, and keeping the walls of the enclosure at about 14° or 15°, found the emissivity in a

Sprengel vacuum, when the surface of the globe was coated with soot, to range from $\cdot 000\ 125$ for an excess of temperature of $83^{\circ} \cdot 6$ to $\cdot 000\ 095$ for an excess of $32^{\circ} \cdot 7$. When the globe was silvered and very highly polished, the emissivity in vacuo ranged from $\cdot 000\ 0354$ for an excess of $72^{\circ} \cdot 1$ to $\cdot 000\ 0296$ for an excess of $43^{\circ} \cdot 7$.

Employing higher temperatures for the globes, but the same temperature of the enclosure, the mean emissivity for the sooted globe was $\cdot 000\ 223$ when the excess was between 230° and 205° , and $\cdot 000\ 122$ when it was between $84^{\circ} \cdot 1$ and $78^{\circ} \cdot 8$.

With dry air at atmospheric pressure in place of vacuum, the emissivity for the sooted globe was multiplied by about $2\frac{1}{2}$, and that for the silvered and polished globe by about 5.

With a platinum wire at 408° as the emitting body, the temperature of the enclosure being about 16° , the emission at the highest vacuum being taken as the unit, the emission was $1\cdot 92$ at a pressure of $\cdot 034$ mm., $3\cdot 12$ at $\cdot 094$ mm., $3\cdot 96$ at $\cdot 14$ mm., $7\cdot 1$ at $\cdot 444$ mm., $9\cdot 2$ at $\cdot 88$ mm., $11\cdot 5$ at $1\cdot 7$ mm., $13\cdot 5$ at $2\cdot 5$ mm., $15\cdot 9$ at 4 mm., $16\cdot 7$ at $5\cdot 7$ mm., $19\cdot 14$ at $17\cdot 2$ mm., $20\cdot 04$ at 42 mm., $21\cdot 00$ at 340 mm., and $21\cdot 48$ at 740 mm.

168. Professor Tait has published (Proc. R. S. E., 1869-70, p. 207) observations by Mr. J. P. Nichol on the loss of heat from blackened and polished copper, in air, at three different pressures, the enclosure being blackened internally and surrounded by water at a temperature of approximately 8° C. When reduced to C.G.S., Professor Tait's Table of Results will stand as follows:—

Pressure 1.014×10^6 [760 millims of mercury].

Blackened.		Bright.	
Temp. Cent.	Loss per sq. cm. per second.	Temp. Cent.	Loss per sq. cm. per second.
61.2	·01746	63.8	·00987
50.2	·01360	57.1	·00862
41.6	·01078	50.5	·00736
34.4	·00860	44.8	·00628
27.3	·00640	40.5	·00562
20.5	·00455	34.2	·00438
		29.6	·00378
		23.3	·00278
		18.6	·00210

Pressure 1.36×10^5 [102 millims. of mercury].

62.5	·01298	67.8	·00492
57.5	·01158	61.1	·00433
53.2	·01048	55	·00383
47.5	·00898	49.7	·00340
43	·00791	44.9	·00302
28.5	·00490	40.8	·00268

Pressure 1.33×10^4 [10 millims. of mercury].

62.5	·01182	65	·00388
57.5	·01074	60	·00355
54.2	·01003	50	·00286
41.7	·00726	40	·00219
37.5	·00639	30	·00157
34	·00569	23.5	·00124
27.5	·00446		
24.2	·00391		

169. *Heat and Energy of Combination with Oxygen.*

1 gramme of	Compound formed.	Gramme- degrees of heat produced.	Equivalent Energy, in Ergs.
Hydrogen,	H ² O	34000 A F	1.43×10^{12}
Carbon,	CO ²	8000 A F	3.36×10^{11}
Sulphur,	SO ²	2300 A F	9.66×10^{10}
Phosphorus,	P ² O ⁵	5747 A	2.41×10^{11}
Zinc,	ZnO	1301 A	5.46×10^{10}
Iron,	Fe ³ O ⁴	1576 A	6.62×10^{10}
Tin,	SnO ²	1233 A	5.18 ,,
Copper,	CuO	602 A	2.53 ,,
Carbonic Oxide, ...	CO ²	2420 A	1.02×10^{11}
Marsh-gas,	CO ² and H ² O	13100 A F	5.50 ,,
Olefiant gas, ...	,,	11900 A F	5.00 ,,
Alcohol,	,,	6900 A F	2.90 ,,

Combustion in Chlorine.

Hydrogen,	HCl	23000 F T	9.66×10^{11}
Potassium,	KCl	2655 A	1.12 „
Zinc,	ZnCl^2	1529 A	6.42×10^{10}
Iron,	Fe^2Cl^6	1745 A	7.33 „
Tin,	SnCl^4	1079 A	4.53 „
Copper,	CuCl^2	961 A	4.04 „

The numbers in the last column are the products of the numbers in the preceding column by 42 millions.

The authorities for these determinations are indicated by the initial letters A (Andrews), F (Favre and Silbermann), T (Thomsen). Where two initial letters are given, the number adopted is intermediate between those obtained by the two experimenters.

170. Difference between the *two specific heats* of a gas.

Let s_1 denote the specific heat of a given gas at constant pressure,

s_2 the specific heat at constant volume,

α the coefficient of expansion per degree,

v the volume of 1 gramme of the gas in cubic centim.
at pressure p dynes per square centim.

When a gramme of the gas is raised from 0° to 1° at the constant pressure p , the heat taken in is s_1 , the increase of volume is αv , and the work done against external resistance is $\alpha v p$ (ergs). This work is the equivalent of the difference between s_1 and s_2 ; that is, we have

$$s_1 - s_2 = \frac{\alpha v p}{J}, \text{ where } J = 4.2 \times 10^7.$$

For dry air at 0° the value of $v p$ is 7.838×10^8 , and α is .003 665. Hence we find $s_1 - s_2 = .0684$. The value of s_1 , according to Regnault, is .2375. Hence the value of s_2 is .1691.

The value of $\frac{s_1 - s_2}{v}$, or $\frac{ap}{J}$, for dry air at 0° and a megadyne per square centim., is

$$\frac{s_1 - s_2}{v} = \frac{.0684}{783.8} = 8.727 \times 10^{-5};$$

and this is also the value of $\frac{s_1 - s_2}{v}$ for any other gas (at the same temperature and pressure) which has the same coefficient of expansion.

171. *Change of freezing point due to change of pressure.*

Let the volume of the substance in the liquid state be to its volume in the solid state as 1 to $1 + e$.

When unit volume in the liquid state solidifies under pressure $P + p$, the work done by the substance is the product of $P + p$ by the increase of volume e , and is therefore $Pe + pe$.

If it afterwards liquefies under pressure P , the work done against the resistance of the substance is Pe ; and if the pressure be now increased to $P + p$, the substance will be in the same state as at first.

Let T be the freezing temperature at pressure P ,
 $T + t$ the freezing temperature at pressure $P + p$,
 l the latent heat of liquefaction,
 d the density of the liquid.

Then d is the mass of the substance, and ld is the heat taken in at the temperature of melting, T . Hence, by thermodynamic principles, the heat converted into mechanical effect in the cycle of operations is

$$\frac{-t}{T + 273} \cdot ld.$$

But the mechanical effect is pe . Hence we have

$$\frac{-t}{T+273}ld = \frac{pe}{J},$$

$$-\frac{t}{p} = \frac{e(T+273)}{Jld} : \quad \dots \quad (3)$$

$-\frac{t}{p}$ is the lowering of the freezing-point for an additional pressure of a dyne per square centim.; and $-\frac{t}{p} \times 10^6$ will be the lowering of the freezing point for each additional atmosphere of 10^6 dynes per square centim.

For water we have

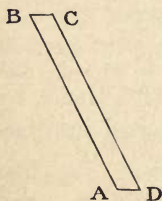
$$e = .087, \quad l = 79.25, \quad T = 0, \quad d = 1,$$

$$-\frac{t}{p} \times 10^6 = \frac{.087 \times 273}{42 \times 79.25} = .00714.$$

Formula (3) shows that $\frac{t}{p}$ is opposite in sign to e . Hence the freezing point will be raised by pressure if the substance contracts in solidifying.

172. Change of *temperature* produced by *adiabatic compression* of a fluid; that is, by compression under such circumstances that no heat enters or leaves the fluid.

Let a cubic centim. of fluid at the initial temperature t° C. and pressure p dynes per square centim. be put through the cycle of operations represented by the annexed "indicator diagram," ABCD, where horizontal distance from left to right denotes increase of volume and perpendicular distance upwards increase of pressure.



In AD let the pressure be constant and equal to p .

In BC let the pressure be constant and equal to $p + \pi$, π being small.

Let AB and CD be adiabatics, so near together that AD and BC are very small compared with the altitude of the figure which is π .

The figure will be ultimately a parallelogram, so that the changes of volume AD and BC will be equal; let their common value be called edt , e denoting the expansion per degree at constant pressure; dt will therefore be the difference of temperature between A and D, or between B and C. We suppose this difference to be very small compared with the difference of temperature between A and B or between C and D.

The cycle is reversible; let it be performed in the direction ABCD. Then heat is taken in as the substance expands from B to C, and given out as it contracts from D to A.

The work done by the substance in the cycle is equal to the area of the parallelogram, which, being the product of the base edt by the height π , is πedt . The heat given out in DA is Cdt , C denoting the thermal capacity of a cubic centim. of the substance at constant pressure; hence the "efficiency" is $\frac{\pi e}{JC}$, and this, by the rules of

Thermodynamics, must be equal to $\frac{\tau}{273+t}$, where τ denotes the increase of temperature from A to B. Put T for the absolute temperature $273+t$, then we have

$$\frac{\tau}{\pi} = \frac{Te}{JC},$$

where τ is the increase of temperature produced by the increase π of pressure.

173. *Elasticity as affected by heat of compression.*

The expansion due to the increase of temperature τ , above calculated, is τe ; that is, $\frac{T\pi e^2}{JC}$; and the ratio of this expansion to the contraction $\frac{\pi}{E}$, which would be produced at constant temperature (E denoting the elasticity of volume at constant temperature), is $\frac{ETe^2}{JC} : 1$. Putting m for $\frac{ETe^2}{JC}$, the elasticity for adiabatic compression will be $\frac{E}{1-m}$; or, if m is small, $E(1+m)$; and this value is to be used instead of E in calculating the change of volume due to sudden compression.

The same formula expresses the value of Young's modulus of elasticity, for sudden extension or compression of a solid in one direction, E now denoting the value of the modulus at constant temperature, and e the coefficient of linear expansion.

Examples.

For compression of water between 10° and 11° we have

$$E = 2.1 \times 10^{10}, T = 283, e = .000\ 092, C = 1;$$

hence
$$\frac{ETe^2}{JC} = .0012.$$

For longitudinal extension of iron at 10° we have

$$E = 1.96 \times 10^{12}, T = 283, e = .000\ 0122, C = .109 \times 7.7;$$

hence
$$\frac{ETe^2}{JC} = .00234.$$

Thus the heat of compression increases the volume-

elasticity of water at this temperature by about $\frac{1}{8}$ per cent., and the longitudinal elasticity of iron by about $\frac{1}{4}$ per cent.

For dry air at 0° and a megadyne per square centim., we have

$$E = 10^6, T = 273, e = \frac{1}{273}, C = \cdot 2375 \times \cdot 001276,$$

$$m = \frac{ETe^2}{JC} = \cdot 288, \frac{1}{1-m} = 1\cdot 404.$$

174. *Expansions of Volume per degree Cent. (abridged from Watts' Dictionary of Chemistry, Article Heat, pp. 67, 68, 71).*

Glass,.....	$\cdot 000\ 02$ to	$\cdot 000\ 03$
Iron,.....	$\cdot 000\ 035$,,	$\cdot 000\ 044$
Copper,.....	$\cdot 000\ 052$,,	$\cdot 000\ 057$
Platinum,.....	$\cdot 000\ 026$,,	$\cdot 000\ 029$
Lead,.....	$\cdot 000\ 084$,,	$\cdot 000\ 089$
Tin,.....	$\cdot 000\ 058$,,	$\cdot 000\ 069$
Zinc,.....	$\cdot 000\ 087$,,	$\cdot 000\ 090$
Gold,.....	$\cdot 000\ 044$,,	$\cdot 000\ 047$
Brass,.....	$\cdot 000\ 053$,,	$\cdot 000\ 056$
Silver,.....	$\cdot 000\ 057$,,	$\cdot 000\ 064$
Steel,.....	$\cdot 000\ 032$,,	$\cdot 000\ 042$
Cast Iron,.....	about	$\cdot 000\ 033$

These results are partly from direct observation, and partly calculated from observed linear expansion.

175. Very exact determinations of the coefficients of linear expansion of certain substances have been made by Benoît at the Bureau International des Poids et Mesures, and are described in tom. VI. of *Travaux et Memoires*, where the chief results are collected at p. 190 In terms

of temperature by hydrogen thermometer, some of them are as follows :—

Quartz, along axis,	$10^{-9} (7161\cdot4 + 8\cdot01 t).$
„ perpendicular to axis,	$10^{-9} (13254\cdot6 + 11\cdot63 t).$
Beryl, along axis,	$10^{-9} (-1347\cdot8 + 4\cdot12 t).$
„ perpendicular to axis,	$10^{-9} (+1002\cdot5 + 4\cdot57 t).$
Iceland-spar, along axis,	$10^{-9} (25135\cdot3 + 11\cdot80 t).$
„ perpendicular to axis,	$10^{-9} (-5578\cdot2 + 1\cdot38 t).$
Platinum, pure,	$10^{-9} (8901 + 1\cdot21 t).$
Iridium, „	$10^{-9} (6358 + 3\cdot21 t).$
Platinum-iridium, containing 10 per cent. of iridium, about,	$10^{-9} (8650 + 1\cdot8 t).$
Steel, tempered,	
„ „ another specimen,	$10^{-9} (10354 + 5\cdot23 t).$
„ „	$10^{-9} (10457 + 5\cdot20 t).$
Bronze, about,	$10^{-9} (17545 + 5\cdot25 t).$

Where we have added the word “about,” the original gives results for various specimens, and we have adopted a mean.

176. *Expansion of Mercury, according to Regnault (Watts' Dictionary, p. 56).*

Temp. = t .	Volume at t° .	Expansion per degree at t° .
0	1·000 000	·000 179 05
10	1·001 792	·000 179 50
20	1·003 590	·000 180 01
30	1·005 393	·000 180 51
50	1·009 013	·000 181 52
70	1·012 655	·000 182 53
100	1·018 153	·000 184 05

The temperatures are by air thermometer.

The formula adopted by the *Bureau International des Poids et Mesures* for the volume at t° C. (derived from Regnault's results) is

$$1 + \cdot000\,181\,792t + \cdot000\,000\,000\,175t^2 \\ + \cdot000\,000\,000\,035\,116t^3.$$

177. *Expansion of certain Liquids, as determined by Kopp and Pierre.*

Temp.	Volume of			
	Alcohol.	Ether.	Bisulphide of Carbon.	Oil of Turpentine.
0°	1.	1.	1.	1.
10	1·01050	1·01518	1·01156	1·00919
20	1·02128	1·03122	1·02350	1·01875
30	1·03242	1·04829	1·03594	1·02865
40	1·04404	1·06654	1·04901	1·03886

178. *Collected Data for Dry Air.*

Expansion from 0° to 100° at const. pressure,	as 1 to 1·367 or as 273 to 373
Specific heat at constant pressure,.....	·2375
„ „ at constant volume,	·1691
Pressure-height at 0° C., about $7\cdot99 \times 10^5$ cm.,	or about 26210 ft.
Standard barometric column,.....	76 cm. = 29·922 inches.
Standard pressure,.....	1033·3 gm. per sq. cm. or 14·7 lbs. per sq. inch. or 2117 lbs. „ foot. or $1\cdot0136 \times 10^6$ dynes per sq. cm.
Standard density, at 0° C.,.....	·001293 gm. per cub. cm. or ·0807 lbs. per cub. foot.
Standard bulkiness,.....	773·3 cub. cm. per gm. or 12·39 cub. ft. per lb.

Dry and Moist Air.

Temp.	Mass of 1 Cubic Metre in Grammes.		
	Dry Air.	Saturated Air.	Vapour at Saturation.
0°	1293·1	1290·2	4·9
10	1247·3	1241·7	9·4
20	1204·6	1194·3	17·1
30	1164·8	1146·8	30·0
40	1127·6	1097·2	50·7

If A denote the density of dry air and W that of vapour at saturation, the density of saturated air is $A - \frac{3}{5}W$, or more exactly $A - \cdot608 W$.

CHAPTER X.

MAGNETISM.

179. THE *unit magnetic pole*, or the pole of *unit strength* is that which repels an equal pole at unit distance with unit force. In the C.G.S. system it is the pole which repels an equal pole, at the distance of 1 centimetre, with a force of 1 dyne.

If P denote the strength of a pole, it will repel an equal pole at the distance L with the force $\frac{P^2}{L^2}$. Hence we have the dimensional equations

$P^2 L^{-2} = \text{force} = M L T^{-2}$, $P^2 = M L^3 T^{-2}$, $P = M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}$; that is, the dimensions of a *pole* (or the dimensions of *strength of pole*) are $M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}$.

180. The work required to move a pole P from one point to another is the product of P by the difference of the *magnetic potentials* of the two points. Hence the dimensions of magnetic potential are

$$\frac{\text{work}}{P} = M L^2 T^{-2}. \quad M^{-\frac{1}{2}} L^{-\frac{3}{2}} T = M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}.$$

181. The *intensity or strength of a magnetic field* is the force which a unit pole will experience when placed in it.

Denoting this intensity by H , the force on a pole P will be HP . Hence

$$HP = \text{force} = MLT^{-2}, H = MLT^{-2} \cdot M^{-\frac{1}{2}}L^{-\frac{3}{2}}T = M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1};$$

that is, the dimensions of *field intensity* are $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$.

182. The *moment* of a *magnet* is the product of the strength of either of its poles by the distance between them. Its dimensions are therefore LP ; that is, $M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}$.

Or, more rigorously, the moment of a magnet is a quantity which, when multiplied by the intensity of a uniform field, gives the couple which the magnet experiences when held with its axis perpendicular to the lines of force in this field. It is therefore the quotient of a couple ML^2T^{-2} by a field-intensity $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$; that is, it is $M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}$ as before.

183. If different portions be cut from a uniformly magnetised substance, their moments will be simply as their volumes. Hence the *intensity of magnetisation* of a uniformly magnetised body is defined as the quotient of its moment by its volume. But we have

$$\frac{\text{moment}}{\text{volume}} = M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1} \cdot L^{-3} = M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}.$$

Hence *intensity of magnetisation* (often called simply *magnetisation*) has the same dimensions as *intensity of field*. When a magnetic substance (whether paramagnetic or diamagnetic) is placed in a magnetic field, it is magnetised by induction. From this point of view the intensity of the field to which the magnetisation is due is called the *magnetising force*.

184. If we suppose a narrow crevasse to be excavated in the magnetised substance, there will be no free mag-

netism on its sides if their direction be *longitudinal*, that is, parallel to the direction of magnetisation; and when we speak of the *magnetising force at a point in a body* we mean the field which would exist in such a crevasse excavated about the point. Magnetising force is now usually denoted by \mathbf{H} . It is called, indifferently, magnetising force, *magnetic force*, or strength of *field*.

On the other hand, if the narrow crevasse be *transverse*, that is, if it cut the lines of magnetisation at right angles, there will be free magnetism of opposite signs on the two faces of the crevasse, the surface-density of this magnetism on either face being numerically equal to the intensity of magnetisation, which is denoted by \mathbf{l} . These two surface-layers produce a field of intensity $4\pi \mathbf{l}$ in the narrow space between them, and this field must be compounded with the field \mathbf{H} in order to obtain the resultant field within the transverse crevasse. This resultant is called the *intensity of induction*, or more briefly the *induction*, at the point in question, and is denoted by \mathbf{B} . Accordingly, whether the body is isotropic or not, \mathbf{B} is the resultant of \mathbf{H} and $4\pi \mathbf{l}$.

185. If the substance is isotropic, and has no magnetism except what is called out by the existing field, \mathbf{H} and \mathbf{l} have parallel directions, which are the same or opposite according as the substance is paramagnetic (like iron) or diamagnetic (like bismuth). In the former case \mathbf{H} and \mathbf{l} must be regarded as having the same sign, and in the latter case opposite signs. In both cases we have the algebraic equation

$$\mathbf{B} = \mathbf{H} + 4\pi \mathbf{l}, \dots \dots \dots (1),$$

which is often written

$$\mu = 1 + 4\pi \kappa, \dots \dots \dots (2),$$

μ denoting the ratio of \mathbf{B} to \mathbf{H} , and κ the ratio of \mathbf{I} to \mathbf{H} . μ is called the *permeability* and κ the *susceptibility* of the substance. These ratios are by no means constant for a given substance, but largely depend on the value of \mathbf{H} . As \mathbf{H} increases from zero, their values (in most cases at least) first increase to a maximum and then decrease. The value of κ when negative is always small: so that μ is always positive, being greater than unity for paramagnetic and less than unity for diamagnetic substances.

The dimensions of \mathbf{B} , \mathbf{H} , and \mathbf{I} are $\mathbf{M}^{\frac{1}{2}} \mathbf{L}^{-\frac{1}{2}} \mathbf{T}^{-1}$; μ and κ are mere numerical quantities independent of the units of mass, length, and time.

186. In air \mathbf{I} is sensibly zero even in strong fields, and \mathbf{B} is therefore sensibly equal to \mathbf{H} .

It can be shown that the component of \mathbf{B} normal to the surface of a magnetised body (whether the magnetisation be temporary or permanent) has the same value just outside as just inside the body; whence it can be deduced that if tubes be drawn following the direction of \mathbf{B} , the value of the product

$\mathbf{B} \times \text{section of tube}$

will be the same at all parts of one and the same tube. Every such tube returns into itself. One portion of it may be within a magnetised body, and the other portion in the external air.

It is convenient to make the tubes of such sizes that the value of the constant product $\mathbf{B} \times \text{section}$ is unity for each. Then the number of these "tubes of induction" that cut any area is called the "flux of induction" across the area. If the area is bounded by a conducting

circuit, any change in the flux of induction will produce in the circuit an electromotive force equal at each instant to the rate of change of this flux.

187. It was formerly thought that the ratios κ and μ were nearly independent of the intensity of the field so long as the magnetisation was far below saturation. This however is very far from being the fact. The following table based upon observations by Ewing* and Bidwell† indicates approximately the changes which occur in the susceptibility and permeability of wrought iron as the strength of the field is increased from a small to a high value. The results obtained with different specimens of iron may vary considerably in details, but the general character of the phenomenon is always the same.

H Magnetising force.	I Magnetisation.	κ Susceptibility.	B Induction.	μ Permeability.
0.3	3	10	41	128
1.4	32	23	413	299
2.2	117	53	1460	670
3.5	574	164	7230	2070
4.9	917	187	11540	2350
6.7	1078	161	13520	2020
10.2	1173	115	14840	1450
22.3	1249	56	15710	705
78	1337	17	16900	215
208	1452	7	18500	89
585	1530	2.6	19800	34
24500	1660	0.067	45300	1.9

* Phil. Trans. 1885, II., p. 541, and 1889, A. p. 226.

† Proc. Roy. Soc., No. 245 (1886), p. 493.

188. When I ceases to increase sensibly for further increase of H , "saturation" is said to be attained. Ewing (Phil. Trans. 1889, A. p. 242) finds that it is attained with magnetising forces of less than 2000 C.G.S. for wrought iron and nickel, and less than 4000 for cast iron and cobalt; the following being the saturation values of I :—

Wrought iron,.....	1700
Cast iron,	1240
Nickel (annealed),.....	515
,, (hard drawn),....	400
Cobalt,.....	1300

189. Magnetic susceptibility and permeability are also affected by such causes as mechanical stress, vibration, and changes of temperature.

In weak magnetic fields the susceptibility of an iron wire is increased by longitudinal tension, whereas in strong fields it may be diminished. The strength of field at which the reversal of the effect of tension takes place is called by Sir W. Thomson the "Villari critical point." It occurs earlier the greater the stress is. The following table, due to Ewing (Phil. Trans., 1885, II., p. 623), gives the critical strength of field and the corresponding magnetisation for a certain iron wire under various loads.

Load—Grms. per Cm.	Critical Field.	Magnetisation.
215000	7·3	1220
430000	4·3	1040
860000	3·4	840
1290000	3·05	690

190. Vibration or jarring appears generally to increase the permeability and susceptibility. With a small magnetising force Ewing has found an apparent permeability of no less than 20,000 in a rod of soft annealed iron kept in a state of vibration (Journ. Inst. Elec. Eng., xix. 41).

191. The permeability of iron for small magnetising forces increases with rise of temperature, slowly at first, and afterwards more rapidly, until the iron is red hot, when it suddenly falls to 1, and the iron becomes non-magnetic. In an experiment by Hopkinson with a magnetising force of 0.3 the permeability of an iron ring was 11,000 at a temperature of 770° , and only 1.14 at a temperature of 785° (Journ. Inst. E. E., xix. 26).

Nickel becomes non-magnetisable at a temperature of about 300° , and cobalt at a bright yellow heat.

Critical Temperature for Iron.

192. In the case of iron, certain other remarkable changes of property occur at the same temperature at which it ceases to be magnetic. If an iron or steel wire is gradually raised to a red heat by an electric current, a spontaneous fall in its temperature and contraction of its length occur momentarily when it first reaches this temperature. When it has been heated higher and is allowed to cool, a still more marked spontaneous rise of temperature called "recalcescence" and momentary elongation occur on arriving at the same temperature. The thermoelectric properties of iron also exhibit a striking irregularity at and near this temperature.

Various Substances.

193. For nickel and cobalt Rowland found that as the

magnetising force was increased, μ increased to a maximum and then diminished, the following being the approximate values of the maxima and of the magnetising forces with which they were obtained:—

	Max. μ	Field.
Nickel at 15° C.,	220	9
„ 220°,	315	5 or 6
Cobalt at 5° C.,	142	18
„ -5°,	144	above 18
„ 230°,	236	11 or 12

194. The following results are selected from those contained in Prof. Chrystal's article "Magnetism" in the Encyc. Brit.

For bismuth, Von Ettingshausen obtained - 14 as the value of $10^6 \kappa$ under magnetising forces ranging from 26 to 110 C.G.S., with some evidence of diminishing value for the larger forces.

For ferric chloride, Silow found a maximum value of $10^6 \kappa$ amounting to about 170, corresponding to a magnetising force .4 C.G.S. The value was only 34 when the magnetising force was .08.

The following values of $10^6 \kappa$ for liquids and gases were obtained by Schuhmeister.

Liquids.

Magnetising force,	61·5	130·8	252·7
Water,	-·55	-·45	-·44
Alcohol,	-·45	-·42	-·38
Bisulphide of carbon,	-·46	..·39	-·37
Ether,	-·40	-·29	...

Gases.

Magnetising force,	66·8	141·8	272·2
Oxygen,	·05	·06	·12
Nitrogen,	·025	·038	·047

For concentrated solutions of salts of iron Plücker obtained the following values of *susceptibility divided by density*, in terms of an arbitrary unit.

Ferrous sulphate,...	126	Ferrous chloride, ...	84
Ferric chloride,	98	Ferric sulphate,	58

the scale being such that iron was 100,000, magnetic iron ore 40,000, specular iron ore 533, and hematite 134.

Changes of Length.

195. The length of a bar of magnetisable metal is in general altered by longitudinal magnetisation. In a continually increasing field, the length of an iron bar is at first increased and afterwards diminished; that of a

Magnetic Field.	Elongations in Ten-millionths of original lengths.		
	Iron.	Cobalt.	Nickel.
65	13		-104
125	19*	-10	-167
237	7	-31	-218
293	0	-37	-233
343	-6	-44†	-240
500	-35	-30	
745	-50	0	
1120	-65	45	
1400	-66	75	-245

* Maximum increment.

† Maximum decrement.

cobalt bar is at first diminished and afterwards increased, while that of a nickel bar is always diminished. The annexed table shows the nature of the positive and negative elongations of certain rods of iron, cobalt and nickel under magnetisation as observed by Bidwell (Phil. Trans., 1888, A. pp. 226-8):—

ELECTROMAGNETISM.

196. A circular current C , of radius r , exerts upon a unit pole placed at its centre a force $\frac{2\pi C}{r}$, the strength of the current being expressed in electromagnetic units. A north pole is urged along the axis of the circuit in a direction which is related to that of the current as the longitudinal to the rotational motion of a right-handed screw.

The intensity of force inside a solenoid or coil, the length of which is great compared with its transverse dimensions, is $4\pi Cn/l$, n being the number of turns of wire, and l the length of the solenoid. The direction of the force may be specified as before. Inside a solenoid forming a hollow circular ring of circular section (like string wound upon an anchor ring) the strength of field is $\frac{2nC}{\rho}$, ρ denoting distance from the rectilinear axis of the ring.

197. An iron core placed within a solenoid becomes inductively magnetised, and to such a combination the term "electromagnet" is specially applied. If an iron core of cross-section A be divided transversely by a very thin film of air, the force in dynes required to separate the two portions can be computed by multiplying the quan-

tity of free magnetism AI on one face by the intensity of force \mathbf{B} in the intervening space minus that part of it—namely, $\frac{1}{2} (\mathbf{B} - \mathbf{H})$ —which is due to the face itself.

This gives $\frac{1}{2} AI (\mathbf{B} + \mathbf{H})$ or $\frac{A}{8\pi} (\mathbf{B}^2 - \mathbf{H}^2)$.

In some experiments made by Bidwell with two semi-circular electromagnets arranged with their ends in contact, a weight of 15905 grammes per square centim. was carried, the observed value of \mathbf{H} being 585, and that of \mathbf{B} , calculated by the above formula, 19820.

Examples.

1. To find the multiplier for reducing magnetic intensities from the foot-grain-second system to the C.G.S. system.

The dimensions of the unit of intensity are $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$. In the present case we have $M = \cdot 0648$, $L = 30\cdot 48$, $T = 1$, since a grain is $\cdot 0648$ gramme, and a foot is $30\cdot 48$ centim.

Hence $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1} = \sqrt{\frac{\cdot 0648}{30\cdot 48}} = \cdot 04611$; that is, the foot-grain-second unit of intensity is denoted by the number $\cdot 04611$ in the C.G.S. system. This number is accordingly the required multiplier.

2. To find the multiplier for reducing intensities from the millimetre-milligramme-second system to the C.G.S. system, we have

$$M = \frac{1}{1000}, \quad L = \frac{1}{10}, \quad T = 1, \quad M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1} = \sqrt{\frac{10}{1000}} = \frac{1}{10}.$$

Hence $\frac{1}{10}$ is the required multiplier.

3. Gauss states (Taylor's Scientific Memoirs, vol. ii. p. 225) that the magnetic moment of a steel bar-magnet, of one pound weight, was found by him to be 100877000 millimetre-milligramme-second units. Find its moment in C.G.S. units.

Here the value of the unit moment employed is, in terms of C.G.S. units, $M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}$, where M is 10^{-3} , L is 10^{-1} , and T is 1; that is, its value is $10^{-\frac{3}{2}} \cdot 10^{-\frac{5}{2}} = 10^{-4}$. Hence the moment of the bar is 10087·7 C.G.S. units.

4. Find the mean intensity of magnetisation of the bar, assuming its specific gravity to be 7·85, and assuming that the pound mentioned in the question is the pound avoirdupois of 453·6 grammes.

Its mass in grammes, divided by its density, will be its volume in cubic centimetres; hence we have

$$\frac{453\cdot6}{7\cdot85} = 57\cdot78 = \text{volume of bar.}$$

$$\text{Intensity of magnetisation} = \frac{\text{moment}}{\text{volume}} = \frac{10088}{57\cdot78} = 174\cdot6.$$

5. Kohlrausch states (Physical Measurements, p. 195, English edition) that the *maximum of permanent magnetism* which very thin rods can retain is about 1000 millimetre-milligramme-second units of moment for each milligramme of steel. Find the corresponding moment per gramme in C.G.S. units, and the corresponding intensity of magnetisation.

For the moment of a milligramme we have $1000 \times 10^{-4} = 10^{-1}$.

For the volume of a milligramme we have $(7\cdot85)^{-1} \times 10^{-3}$, taking 7·85 as the density of steel.

Hence the moment per gramme is $10^{-1} \times 10^3 = 100$, and the intensity of magnetisation is $100 \times 7.85 = 785$.

6. Gauss states (*loc. cit.*) that the magnetic moment of the earth, in millimetre-milligramme-second measure, is

$$3.3092 R^3,$$

R denoting the earth's radius in millimetres. Reduce this value to C.G.S.

Since R^3 is of the dimensions of volume, the other factor, 3.3092, must be of the dimensions of intensity. Hence, employing the reducing factor 10^{-1} above found, we have .33092 as the corresponding factor for C.G.S. measure; and the moment of the earth will be

$$.33092 R^3,$$

R denoting the earth's radius in centimetres—that is 6.37×10^8 .

We have

$$.33092 \times (6.37 \times 10^8)^3 = 8.55 \times 10^{25}$$

for the earth's magnetic moment in C.G.S. units.

7. From the above result, deduce the intensity of magnetisation of the earth regarded as a uniformly magnetised body.

We have

$$\text{intensity} = \frac{\text{moment}}{\text{volume}} = \frac{8.55 \times 10^{25}}{1.083 \times 10^{27}} = .0790.$$

This is about $\frac{1}{2200}$ of the intensity of magnetisation of

Gauss's pound magnet; so that 2.2 cubic decimetres of earth would be equivalent to 1 cubic centim. of strongly magnetized steel, if the observed effects of terrestrial magnetism were due to uniform magnetization of the earth's substance.

8. Gauss, in his papers on terrestrial magnetism, em-

employs two different units of intensity, and makes mention of a third as “the unit in common use.” The relation between them is pointed out in the passage above referred to. The total intensity at Göttingen, for the 19th of July, 1834, was 4.7414 when expressed in terms of one of these units—the millimetre-milligramme-second unit; was 1357 when expressed in terms of the other unit employed by Gauss, and 1.357 in terms of “the unit in common use.” In C.G.S. measure it would be .47414.

198. A first approximation to the *distribution* of magnetic force *over the earth's surface* is obtained by assuming the earth to be uniformly magnetized, or, what is mathematically equivalent to this, by assuming the observed effects to be due to a small magnet at the earth's centre. The moment of the earth on the former supposition, or the moment of the small magnet on the latter, must be

$$.33092 R^3,$$

R denoting the earth's radius in centims. The magnetic poles, on these suppositions, must be placed at

$$77^\circ 50' \text{ north lat., } 296^\circ 29' \text{ east long.,}$$

and at $77^\circ 50' \text{ south lat., } 116^\circ 29' \text{ east long.}$

The intensity of the horizontal component of terrestrial magnetism, at a place distant A° from either of these poles, will be

$$.33092 \sin A^\circ;$$

the intensity of the vertical component will be

$$.66184 \cos A^\circ;$$

and the tangent of the dip will be

$$2 \cotan A^\circ.$$

The magnetic potential, on the same supposition, will be

$$.33092 \frac{R^3}{r^2} \cos A^\circ,$$

r being variable. (See Maxwell, Electricity and Magnetism, vol. ii. p. 8.) Gauss's approximate expression for the potential and intensity at an arbitrary point on the earth's surface consists of four successive approximations, of which this is the first.

199. According to Airy on Magnetism, the place of greatest horizontal intensity is in lat. 0° long., 259° E., where the value is $\cdot 3733$; the place of greatest total intensity is in South Victoria, about 70° S., 160° E., where its value is $\cdot 7898$, and the place of least total intensity is near St. Helena, in lat. 16° S., long. 355° E., where its value is $\cdot 2828$.

200. The following are the values of the principal magnetic elements at a number of stations in the British Isles, for Jan. 1, 1886, according to a recent survey by Professors Rücker and Thorpe (Bakerian Lecture, Phil. Trans., 1890). They are arranged in order of latitude.

Great Britain and Small Islands.

	Decl.		Dip.		Hor. Force.	Vert. Force.
Lerwick,.....	20	30	72	47	$\cdot 1471$	$\cdot 4748$
Wick,.....	21	15	72	10	$\cdot 1514$	$\cdot 4706$
Stornoway,.....	24	16	72	10	$\cdot 1520$	$\cdot 4726$
Inverness,.....	21	43	71	31	$\cdot 1564$	$\cdot 4680$
Aberdeen,.....	20	16	71	12	$\cdot 1573$	$\cdot 4623$
Dundee,.....	20	44	70	52	$\cdot 1600$	$\cdot 4613$
Iona,.....	23	29	70	56	$\cdot 1618$	$\cdot 4671$
Edinburgh,.....	20	47	70	38	$\cdot 1618$	$\cdot 4606$
Glasgow,.....	21	12	70	45	$\cdot 1606$	$\cdot 4600$
Berwick,.....	19	36	70	16	$\cdot 1648$	$\cdot 4595$
Campbeltown,.....	22	8	70	34	$\cdot 1624$	$\cdot 4605$
Newcastle,.....	19	30	69	50	$\cdot 1666$	$\cdot 4536$
Stranraer,.....	21	35	70	14	$\cdot 1644$	$\cdot 4571$

	Decl.		Dip.		Hor. Force.	Vert. Force.
Carlisle,	20	26	69	54	·1662	·4543
Ramsey,	20	55	69	55	·1662	·4545
Barrow,	20	9	69	31	·1688	·4516
Leeds, ..	19	9	69	11	·1708	·4492
Hull,	18	58	69	4	·1712	·4476
Manchester,	19	17	69	4	·1712	·4476
Birkenhead,	19	58	69	4	·1718	·4491
Holyhead,	20	51	69	23	·1696	·4508
Nottingham,	18	45	68	38	·1747	·4464
Cromer,	17	36	68	20	·1760	·4431
Shrewsbury,	19	41	68	36	·1734	·4427
Birmingham,	18	44	68	21	·1767	·4452
Cambridge,	18	5	68	2	·1778	·4410
Harwich,	17	19	67	38	·1803	·4383
Oxford,	18	34	67	58	·1789	·4419
Milford,	20	9	68	10	·1781	·4444
Cardiff,	19	20	67	52	·1794	·4413
Greenwich,						
Kew,	18	16	67	37	·1809	·4395
Dover,	16	57	67	8	·1834	·4348
Ryde,	18	2	67	8	·1839	·4360
Exeter,	19	29	67	26	·1826	·4395
Falmouth,	19	53	67	15	·1832	·4370
Alderney,	18	3	66	39	·1869	·4329

IRELAND.

Londonderry,	22	50	70	27	·1634	·4560
Bangor,	21	44	70	1	·1660	·4566
Armagh,	22	16	69	58	·1662	·4558
Sligo,	23	5	70	18	·1643	·4588
Dublin,	21	41	69	16	·1709	·4513
Galway,	24	6	69	53	·1658	·4527
Parsonstown,	22	27	69	30	·1699	·4545
Wexford,	21	18	68	56	·1732	·4498
Cork,	22	18	68	46	·1751	·4507
Valentia,	23	16	68	55	·1745	·4524

Let x denote the excess of the latitude (in degrees) above 51° , and let y denote west longitude (in degrees); the annual changes in the above values are :—

For Declination, ... $-(6\cdot2 + \cdot2x + \cdot15y)$ minutes.
 For Dip, ... $-(1\cdot6 - \cdot09x + \cdot04y)$ minutes.
 For Horizontal Force, $+(\cdot00019 - \cdot000\ 006x + \cdot000\ 003y)$.

See table, page 325 of Bakerian Lecture.

201. The following mean values of the magnetic elements at Greenwich have been kindly furnished by the Astronomer Royal :—

West Declination, ... $17^\circ\ 41'\cdot5 - 6'\cdot75 \times (t - 1888)$
 Dip, ... $67^\circ\ 25'\cdot3 - 1'\cdot29 \times (t - 1888)$
 Horizontal Force, ... $0\cdot18196 + \cdot000\ 187 \times (t - 1888)$
 Vertical Force, ... $0\cdot43760 - \cdot000\ 013 \times (t - 1888)$
 $= \text{Horizontal Force} \times \tan. \text{Dip}.$

Each of these formulæ gives the mean of the entire year t .

202. According to Lord Rayleigh's determination (Phil. Trans., 1885, II. p. 343) the rotation of the plane of polarisation between two points, 1 centim. apart, in bisulphide of carbon at 18° C., whose magnetic potentials (in C.G.S. measure) differ by unity, is $\cdot04202$ minute, or $1\cdot22231 \times 10^{-5}$ radian, for sodium light. At 0° C. the rotation is

according to H. Becquerel, $\cdot04341$ min.
 „ J. E. H. Gordon, $\cdot0433$ „
 „ L. Arons, $\cdot0439$ „

CHAPTER XI.

ELECTRICITY.

Electrostatics.

203. If q denote the numerical value of a *quantity* of electricity in electrostatic measure, the mutual force between two equal quantities q at the mutual distance l will be $\frac{q^2}{l^2}$. In the C.G.S. system the electrostatic unit of electricity is accordingly that quantity which would repel an equal quantity at the distance of 1 centim. with a force of 1 dyne.

Since the dimensions of force are $\frac{ml}{t^2}$, we have, as regards dimensions,

$$\frac{q^2}{l^2} = \frac{ml}{t^2}, \text{ whence } q^2 = \frac{ml^3}{t^2}, \quad q = m^{\frac{1}{2}} l^{\frac{3}{2}} t^{-1}.$$

204. The work done in raising a quantity of electricity q through a difference of *potential* v is qv .

Hence we have

$$v = \frac{\text{work}}{q} = ml^2 t^{-2} \cdot m^{-\frac{1}{2}} l^{-\frac{3}{2}} t = m^{\frac{1}{2}} l^{\frac{1}{2}} t^{-1}.$$

In the C.G.S. system the unit difference of potential is that difference through which a unit of electricity must be raised that the work done may be 1 erg.

Or, we may define potential as the quotient of quantity of electricity by distance. This gives

$$v = m^{\frac{1}{2}} l^{\frac{3}{2}} t^{-1} \cdot l^{-1} = m^{\frac{1}{2}} l^{\frac{1}{2}} t^{-1}, \text{ as before.}$$

In the C.G.S. system the unit of potential is the potential due to unit quantity at the distance of 1 centim.

205. The *capacity* of a conductor is the quotient of the quantity of electricity with which it is charged by the potential which this charge produces in it. Hence we have

$$\text{capacity} = \frac{q}{v} = m^{\frac{1}{2}} l^{\frac{3}{2}} t^{-1} \cdot m^{-\frac{1}{2}} l^{-\frac{1}{2}} t = l.$$

The same conclusion might have been deduced from the fact that the capacity of an isolated spherical conductor is equal (in numerical value) to its radius. The C.G.S. unit of capacity is the capacity of an isolated sphere of 1 centim. radius.

206. The numerical value of a *current* (or the strength of a current) is the quantity of electricity that passes in unit time.

Hence the dimensions of *current* are $\frac{q}{t}$; that is, $m^{\frac{1}{2}} l^{\frac{3}{2}} t^{-2}$.

The C.G.S. unit of current is that current which conveys the above defined unit of quantity in 1 second.

207. The dimensions of *resistance* can be deduced from Ohm's law, which asserts that the resistance of a wire is the quotient of the difference of potential of its two ends, by the current which passes through it. Hence we have

$$\text{resistance} = m^{\frac{1}{2}} l^{\frac{3}{2}} t^{-1} \cdot m^{-\frac{1}{2}} l^{-\frac{3}{2}} t^2 = l^{-1} t.$$

Or, the resistance of a conductor is equal to the time required for the passage of a unit of electricity through it,

when unit difference of potential is maintained between its ends. Hence

$$\text{resistance} = \frac{\text{time} \times \text{potential}}{\text{quantity}} = t \cdot m^{\frac{1}{2}} l^{\frac{1}{2}} t^{-1} \cdot m^{-\frac{1}{2}} l^{-\frac{3}{2}} t = l^{-1} t.$$

208. As the force upon a quantity q of electricity, in a field of electrical force of intensity i , is iq , we have

$$i = \frac{\text{force}}{q} = m l t^{-2} \cdot m^{-\frac{1}{2}} l^{-\frac{3}{2}} t = \bar{m}^{\frac{1}{2}} l^{-\frac{1}{2}} t^{-1}.$$

The quantity here denoted by i is commonly called the "electrical force at a point."

Electromagnetics.

209. A *current* C (or a current of *strength* C) flowing along a circular arc, produces at the centre of the circle an intensity of magnetic field equal to C multiplied by length of arc divided by square of radius. Hence C divided by a length is equal to a field-intensity, or

$$C = \text{length} \times \text{intensity} = L \cdot M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1} = L^{\frac{1}{2}} M^{\frac{1}{2}} T^{-1}.$$

210. The *quantity* of electricity Q conveyed by a current is the product of the current by the time that it lasts. The dimensions of Q are therefore $L^{\frac{1}{2}} M^{\frac{1}{2}} T$.

211. The work done in urging a quantity Q through a circuit, by an *electromotive force* E , is EQ ; and the work done in urging a quantity Q through a conductor, by means of a *difference of potential* E between its ends, is EQ . Hence the dimensions of electromotive force, and also the dimensions of *potential*, are $ML^2T^{-2} \cdot L^{-\frac{1}{2}} M^{-\frac{1}{2}}$, or $M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2}$.

212. The *capacity* of a conductor is the quotient of quantity of electricity by potential. Its dimensions are therefore

$$M^{\frac{1}{2}}L^{\frac{1}{2}} \cdot M^{-\frac{1}{2}}L^{-\frac{3}{2}}T^2; \text{ that is, } L^{-1}T^2.$$

213. Resistance is $\frac{E}{C}$; its dimensions are therefore

$$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2} \cdot M^{-\frac{1}{2}}L^{-\frac{1}{2}}T; \text{ that is, } LT^{-1}.$$

214. The following table exhibits the dimensions of each electrical element in the two systems, together with their ratios :—

	Dimensions in Electrostatic System.	Dimensions in Electromagnetic System.	Dimensions in E.S. Dimensions in E.M.
Quantity,	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$	$M^{\frac{1}{2}}L^{\frac{1}{2}}$	LT^{-1}
Current,	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$	LT^{-1}
Capacity,	L	$L^{-1}T^2$	L^2T^{-2}
Potential and electromotive force, }	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$	$L^{-1}T$
Resistance,	$L^{-1}T$	LT^{-1}	$L^{-2}T^2$

215. The *heat generated* in time T by the passage of a current C through a wire of resistance R (when there are no Peltier or Thomson effects in the wire) is $\frac{C^2RT}{J}$

gramme degrees, J denoting 4.2×10^7 ; and this is true whether C and R are expressed in electromagnetic or in electrostatic units.

Ratios of the two sets of Electric Units.

216. Let us consider any general system of units based on

a unit of length equal to L centims.,

a unit of mass equal to M grammes,

a unit of time equal to T seconds.

Then we shall have the electrostatic unit of quantity equal to

$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$ C.G.S. electrostatic units of quantity,
and the electromagnetic unit of quantity equal to

$M^{\frac{1}{2}}L^{\frac{1}{2}}$ C.G.S. electromagnetic units of quantity.

It is possible so to select L and T that the electrostatic unit of quantity shall be equal to the electromagnetic unit. We shall then have (dividing out by $M^{\frac{1}{2}}L^{\frac{1}{2}}$)

LT^{-1} C.G.S. electrostatic units

$= 1$ C.G.S. electromagnetic unit;

or the ratio of the C.G.S. electromagnetic unit to the

C.G.S. electrostatic unit is $\frac{L}{T}$.

Now $\frac{L}{T}$ is clearly the value in centims. per second of that velocity which would be denoted by unity in the new system. This is a *definite concrete velocity*; and its numerical value will always be equal to the ratio of the electromagnetic to the electrostatic unit of quantity, whatever units of length, mass, and time are employed.

217. It will be observed that the ratio of the two units of quantity is the inverse ratio of their dimensions; and the same can be proved in the same way of the other four electrical elements. The last column of the above table shows that M does not enter into any of the

ratios, and that L and T enter with equal and opposite indices, showing that all the ratios depend only on the velocity $\frac{L}{T}$.

Thus, if the concrete velocity $\frac{L}{T}$ be a velocity of v centims. per second, the following relations will subsist between the C.G.S. units :—

1 electromagnetic unit of quantity = v electrostatic units.

1 „ current = v „

1 „ capacity = v^2 „

v electromagnetic units of potential = 1 electrostatic unit.

v^2 „ resistance = 1 „

218. The latest determination of the value of v is that of Prof. J. J. Thomson and G. F. C. Searle (Phil. Trans., 1890, p. 620), and is 2.9955×10^{10} centims. per second. On page 621 of the same paper is given (as a quotation from a paper by E. B. Rosa) the following list of previous determinations :—

1856	Weber and Kohlrausch, ...	3.107×10^{10}
1869	W. Thomson and King,	2.808 „
1868	Maxwell,	2.842 „
1872	M'Kichan,	2.896 „
1879	Ayrton and Perry,	2.960 „
1880	Shida,	2.955 „
1883	J. J. Thomson,	2.963 „
1884	Klemenčič,	3.019 „
1888	Himstedt,	3.009 „
1889	W. Thomson,	3.004 „
1889	E. B. Rosa,	2.999 „

All these values agree closely with the velocity of light

in vacuo, of which the best determinations are, some of them a little less, and some a little greater than

$$3 \times 10^{10}.$$

We shall adopt this round number as the value of v .

219. The dimensions of the electric units are rather simpler when expressed in terms of length, *density*, and time.

Putting D for density, we have $M = L^3D$. Making this substitution for M , in the expressions above obtained (§ 214), we have the following results:—

	Electrostatic.		Electromagnetic.
Quantity,	$D^{\frac{1}{2}}L^3T^{-1}$	$D^{\frac{1}{2}}L^2$
Current,.....	$D^{\frac{1}{2}}L^3T^{-2}$	$D^{\frac{1}{2}}L^2T^{-1}$
Capacity,	L	$L^{-1}T^2$
Potential,.....	$D^{\frac{1}{2}}L^3T^{-1}$	$D^{\frac{1}{2}}L^3T^{-2}$
Resistance,.....	$L^{-1}T$	LT^{-1}

It will be noted that the exponents of L and T in these expressions are free from fractions.

Specific Inductive Capacity.

220. The specific inductive capacity of an insulating substance is the ratio of the capacity of a condenser in which this substance is the dielectric to that of a condenser in other respects equal and similar in which air is the dielectric. It is of zero dimensions, and its value exceeds unity for all solid and liquid insulators.

According to Maxwell's electromagnetic theory of light, the square root of the specific inductive capacity is equal to the index of refraction for the rays of longest wave-length.

Messrs. Gibson and Barclay, by experiments performed in Sir W. Thomson's laboratory (Phil. Trans., 1871,

p. 573), determined the specific inductive capacity of solid paraffin to be 1.977.

Dr. J. Hopkinson (Phil. Trans., 1877, p. 23) gives the following results of his experiments on different kinds of flint glass :—

Kind of Flint Glass.	Density.	Specific Inductive Capacity.	Quotient by Density.	Index of Refraction for D line.
Very light,.....	2.87	6.57	2.29	1.541
Light,.....	3.2	6.85	2.14	1.574
Dense,.....	3.66	7.4	2.02	1.622
Double extra } dense, }	4.5	10.1	2.25	1.710

In a later series of experiments (Phil. Trans., 1881, Dec. 16), Dr. Hopkinson obtains the following mean determinations :—

Glass.	Specific Inductive Capacity.	Density.	Specific Inductive Capacity.
Hard crown,.....	6.96	2.485	Paraffin, 2.29
Very light flint,.....	6.61	2.87	
Light flint,.....	6.72	3.2	
Dense flint,.....	7.38	3.66	
Double extra-dense flint,	9.90	4.5	
Plate,.....	8.45	—	

221. For liquids Dr. Hopkinson (Proc. Roy. Soc., Jan. 27, 1881) gives the following value of μ_{∞}^2 (computed) and K (observed), K denoting the specific inductive capacity and μ_{∞} the index of refraction for very long waves deduced by the formula

$$\mu = \mu_{\infty} + \frac{b}{\lambda^2},$$

where b is a constant.

	μ_{∞}^2	K
Petroleum spirit (Field's),.....	1·922	1·92
Petroleum oil (Field's),.....	2·075	2·07
„ (common),.....	2·078	2·10
Ozokerit lubricating oil (Field's),.....	2·086	2·13
Turpentine (commercial),.....	2·128	2·23
Castor oil,.....	2·153	4·78
Sperm oil,.....	2·135	3·02
Olive oil,.....	2·131	3·16
Neatsfoot oil,.....	2·125	3·07

This list shows that the equality of μ_{∞}^2 to K (which Maxwell's theory requires) holds nearly true for hydrocarbons, but not for animal and vegetable oils.

222. Wüllner (Sitzungsber. königl. bayer. Akad., March, 1877) finds the following values of specific inductive capacity :—

Paraffin,.....	1·96	Shellac,....	2·95 to 3·73
Ebonite,.....	2·56	Sulphur,...	2·88 to 3·21
Plate glass,.....	6·10		

Boltzmann (Carl's Repertorium, x. 92—165) finds the following values :—

Paraffin,.....	2·32	Colophonium,.....	2·55
Ebonite,.....	3·15	Sulphur,.....	3·84

Schiller (Pogg. Ann., clii. 535, 1874) finds :—

Paraffin,.....	1·85 to 2·47	Caoutchouc,.....	2·12 to 2·34
Ebonite,.....	2·21 to 2·76	Do., vulcanized,	2·69 to 2·94
Plate glass,.	5·83 to 6·34		

Silow (Pogg. Ann., clvi. and clviii.) finds the following values for liquids :—

Oil of turpentine,.....	2·155 to 2·221
Benzene,	2·199
Petroleum,	2·039 to 2·071

Boltzmann (Wien. Akad. Ber. (2), lxx. 342, 1874) finds for sulphur in directions parallel to the three principal axes, the values

$$4.773, \quad 3.970, \quad 3.811.$$

223. Quincke has investigated the specific inductive capacity of certain liquids by two independent methods, namely, by measuring the capacity of a condenser and by measuring the mutual attraction of the two parallel plates which composed the condenser. According to Maxwell's theory, the charging of a condenser produces tension (or diminution of pressure) in the dielectric along the lines of force, and repulsion (or increase of pressure) perpendicular to the lines of force, the tension and repulsion (per unit of cross section) being each equal to

$$\frac{K(A-B)^2}{8\pi c^2},$$

K denoting the specific inductive capacity of the dielectric c the distance between the two parallel plates of the condenser, and $A - B$ their difference of potentials. The following are Quincke's latest determinations as given in the R. S. Proc., 1886, Dec. 16, p. 459 :—

	Ether.	Bisulphide of Carbon.	Benzol.	Rock-Oil.
By capacity,.....	4.211	2.574	2.359	2.025
By attraction, ...	4.394	2.582	2.360	2.073

224. Professors Ayrton and Perry have found the following values of the specific inductive capacities of gases, air being taken as the standard :—

Air,	1.0000		Hydrogen,	0.9998
Vacuum,	0.9985		Coal gas,	1.0004
Carbonic acid, ...	1.0008		Sulphurous acid,	1.0037

Practical Units.

225. The unit of resistance chiefly employed by practical electricians is the *Ohm*, which is theoretically defined as

10^9 C.G.S. electromagnetic units of resistance.

The practical unit of electromotive force is the *Volt*, which is defined as

10^8 C.G.S. electromagnetic units of potential.

The practical unit of current is the *Ampere*. It is defined as

$\frac{1}{10}$ of the C.G.S. electromagnetic unit current, or as the current produced by 1 volt through 1 ohm.

The practical unit of quantity of electricity is the *Coulomb*. It is defined as

$\frac{1}{10}$ of the C.G.S. electromagnetic unit of quantity, or as the quantity conveyed by 1 ampere in 1 second.

The practical unit of capacity is the *Farad*.* It is defined as

10^{-9} of the C.G.S. electromagnetic unit of capacity, or as the capacity of a condenser which holds 1 coulomb when charged to 1 volt.

The practical unit of work employed in connection with these is the *Joule*. It is defined as

10^7 ergs,

or as the work done in 1 second by a current of 1 ampere in flowing through a resistance of 1 ohm.

* As the farad is much too large for practical convenience, its millionth part, called the *microfarad*, is practically employed, and condensers are in use having capacities of a microfarad, and its decimal subdivisions. The microfarad is 10^{-15} of the C.G.S. electromagnetic unit of capacity.

The corresponding practical unit of rate of working is the *Watt*. It is defined as

10^7 ergs per second,

or as the rate at which work is done by 1 ampere flowing through 1 ohm.

Electric Spark.

226. Sir W. Thomson has observed the length of spark between two parallel conducting surfaces maintained at known differences of potential, and has computed the corresponding intensities of electric force by dividing (in each case) the difference of potential by the distance, since the variation of potential per unit distance measured in any direction is always equal to the intensity of the force in that direction. His results, as given on page 258 of *Papers on Electrostatics and Magnetism*, form the first two columns of the following table:—

Distance between Surfaces.	Intensity of force in Electrostatic Units.	Difference of Potential between Surfaces.	
		In Electrostatic Units.	In Electromagnetic Units.
·0086	267·1	2·30	$6·90 \times 10^{10}$
·0127	257·0	3·26	9·78 „
·0127	262·2	3·33	9·99 „
·0190	224·2	4·26	12·78 „
·0281	200·6	5·64	16·92 „
·0408	151·5	6·18	18·54 „
·0563	144·1	8·11	24·33 „
·0584	139·6	8·15	24·45 „
·0688	140·8	9·69	29·07 „
·0904	134·9	12·20	36·60 „
·1056	132·1	13·95	41·85 „
·1325	131·0	17·36	52·08 „

The numbers in the third column are the products of those in the first and second. The numbers in the fourth column are the products of those in the third by 3×10^{10} .

227. Dr. Warren De La Rue and Dr. Hugo W. Müller (Phil. Trans., 1877) have measured the striking distance between the terminals of a battery of chloride of silver cells, the number of cells being sometimes as great as 11000, and the electromotive force of each being 1·03 volt. Terminals of various forms were employed; and the results obtained with parallel planes as terminals have been specially revised by Dr. De La Rue for the present work. These revised results (which were obtained by graphical projection of the actual observations on a larger scale than that employed for the Paper in the Philosophical Transactions) are given below, together with the data from which they were deduced:—

DATA.

No. of Cells.	Striking Distances.	
	In Inches.	In Centims.
1200	0·012	0·0305
2400	·021	·0533
3600	·033	·0838
4800	·049	·1245
5880	·058	·1473
6960	·073	·1854
8040	·088	·2236
9540	·110	·2794
11000	·133	·3378

DEDUCTIONS.

Electromotive Force in Volts.	Striking Distance in Centims.	Volts per Centim.	Intensity of Force in C.G.S. units.	
			Electromagnetic.	Electro- static.
1000	·0205	48770	$4\cdot88 \times 10^{12}$	163
2000	·0430	46500	4·65	155
3000	·0660	45450	4·55	152
4000	·0914	43770	4·38	146
5000	·1176	42510	4·25	142
6000	·1473	40740	4·07	136
7000	·1800	38890	3·89	130
8000	·2146	37280	3·73	124
9000	·2495	36070	3·61	120
10000	·2863	34920	3·49	116
11000	·3245	33900	3·39	113
11330	·3378	33460	3·35	112

Resistance.

228. The resistance of a wire (or more generally of a prism or cylinder) of given material varies directly as its length, and inversely as its cross section. It is therefore equal to

$$R \frac{\text{length}}{\text{section}},$$

where R is a coefficient depending only on the material. R is called the *specific resistance* of the material. Its reciprocal $1/R$ is called the *specific conductivity* of the material.

R is obviously the resistance between two opposite faces of a unit cube of the substance. Hence in the C.G.S. system it is the resistance between two opposite faces of a cubic centim. (supposed to have the form of a cube).

The dimensions of specific resistance are resistance \times length; that is, in electromagnetic measure, velocity \times length; that is L^2T^{-1} .

229. The standard resistance-coils originally issued in 1865 as representing what is now called the ohm, were constructed under the direction of a Committee of the British Association, and their resistance was generally called the *B.A. unit*. Subsequent determinations show that it was too small; and the value adopted for it in the Report of the Electrical Standards Committee of the British Association for 1890 is $\cdot 9866$ of an ohm. Its most probable value to five figures is $\cdot 98656$.

In accordance with a resolution of the International Congress of Electricians, 1881, special attention has been directed to determining the precise length of a column of pure mercury at 0°C. with a section of 1 square millimetre which has a resistance of one ohm. The value adopted for it in the 1890 Report above mentioned, and since adopted by the Board of Trade, is 106.3 centimetres; whence it will follow that the specific resistance of mercury at 0° , correct to four figures, is 94070 C.G.S.

An earlier unit of resistance in use among electricians was Siemens' unit, defined as the resistance of a column of mercury of the same temperature and section as above and 1 metre long. Its value, as deduced from the above length 106.3, is the reciprocal of 1.063, that is $\cdot 9407$ of an ohm. Comparing this with $\cdot 98656$, we obtain $\cdot 9535$ as the ratio of Siemens' unit to the original B.A. standards.

230. The following are some of the principal individual

determinations upon which the adopted length 106·3 is based :—

Rayleigh, 1882,.....	Rotating coil,.....	106·24
„ 1883,.....	Lorenz method,	106·21
G. Wiedemann, 1884,	Rotation through 180°,....	106·19
Mascart, 1884,.....	Induced current,.....	106·33
Rowland, 1887,	Mean of several methods,	106·32
Kohlrausch, 1887,.....	Damping of magnets,	106·32
Glazebrook, 1882 and 1888,	Induced currents,	106·29
Wuilleumeier, 1890,.....	106·27
V. Jones,	Lorenz method,	106·31

231. By way of assisting the memory, it is useful to remark that the numerical value of the *ohm* is the same as the numerical value of a velocity of one *earth-quadrant per second*, since the length of a quadrant of the meridian is 10^9 centims. This equality will subsist whatever fundamental units are employed, since the dimensions of resistance are the same as the dimensions of velocity.

No special names have as yet been assigned to any electrostatic units.

232. The following table of resistances of metals at 0°C ., is derived from Mathiessen's results as given by Jenkin, by changing the specific resistance of mercury from 96190 to 94070 and changing all the other resistances in the same ratio.

	Specific Resistance.	Percentage of Variation for a degree at 20°C .	Resistance in Ohms of a wire. 1 mm. diam. 1 m. long.
Silver, annealed,.....	1488	·377	·0189
„ hard-drawn,.....	1616		·0206
Copper, annealed,	1580	·388	·0201
„ hard-drawn,.....	1616		·0206
Gold, annealed,	2036	·365	·0259
„ hard-drawn,.....	2072		·0264
Aluminium, annealed,.....	2881		·0367

	Specific Resistance.	Percentage of Variation for a degree at 20° C.	Resistance in Ohms of a wire. 1 mm. diam. 1 m. long.
Zinc, pressed,	5566	·365	·0709
Platinum, annealed,	8957		·1140
Iron, annealed,	9611		·1223
Nickel, annealed,	12320		·1569
Tin, pressed,	13070	·365	·1664
Lead, pressed,	19420	·387	·2470
Antimony, pressed,	35110	·389	·4471
Bismuth, pressed,	12970	·354	1·6520
Mercury, liquid,	94070	·072	1·1977
Alloy, 2 parts platinum, 1 part silver, by weight, hard or annealed,	2412	·031	·0307
German silver, hard or annealed,	20710	·044	·2636
Alloy, 2 parts gold, 1 silver, by weight, hard or annealed, }	10750	·065	·1368

*Resistances of Conductors of Telegraphic Cables per
nautical mile, at 24° C., in C.G.S. units.*

Red Sea,	$7\cdot79 \times 10^9$
Malta-Alexandria, mean,	3·42 ,,
Persian Gulf, mean,	6·17 ,,
Second Atlantic, mean,	4·19 ,,

233. The effect of temperature on the resistance of a platinum wire is very fully discussed by Callendar (Phil. Trans., 1887, A, p. 161) and he adopts the exponential formula

$$e^{\frac{0\cdot003\ 4259\ t}{1 + 0\cdot001\ 529\ t}}$$

as best representing the ratio of the resistance of a wire of pure platinum at t° to its resistance at 0° . This makes the resistance at temperatures between 0° and 100° , proportional to $1 + 0\cdot0034\ t$. He states (p. 162) that for

different specimens of commercial platinum the ratio of the resistance at 100° to the resistance at 0° varies from 1.25 to 1.35.

234. The following formulæ are given by Benoît* for the ratio of the specific resistance at t° C. to that at 0° C.:—

Aluminium,	$1 + \cdot 003\ 876\ t + \cdot 000\ 001\ 320\ t^2$
Copper,	$1 + \cdot 003\ 67\ t + \cdot 000\ 000\ 587\ t^2$
Iron,	$1 + \cdot 004\ 516\ t + \cdot 000\ 005\ 828\ t^2$
Magnesium,	$1 + \cdot 003\ 870\ t + \cdot 000\ 000\ 863\ t^2$
Silver,	$1 + \cdot 003\ 972\ t + \cdot 000\ 000\ 687\ t^2$
Tin,	$1 + \cdot 004\ 028\ t + \cdot 000\ 005\ 826\ t^2$

Adopting the formula $1 + at$ for the ratio of the specific resistance at t° to that at 0° , MM. Cailletet and Bouty (Jour. de Phys., July, 1885) have made the following determinations of the coefficient of variation a at very low temperatures:—

	Range of Temperature.		Coefficient of Variation.
Aluminium,	$+28^{\circ}$ to -91°	$\cdot 003\ 85$
Copper,	-23 to -123	$\cdot 004\ 23$
Iron,	0 to -92	$\cdot 004\ 9$
Magnesium,	0 to -88	$\cdot 003\ 90$
Mercury,	-40 to -92	$\cdot 004\ 07$
Silver,	$+30$ to -102	$\cdot 003\ 85$
Tin,	0 to -85	$\cdot 004\ 24$

The new alloy called *platinoid* (consisting of German silver with a little tungsten) was found by Mr. J. T. Bottomley (Proc. Roy. Soc., May 7, 1885) to have an average variation of resistance with temperature of only $\cdot 00022$ per degree centigrade, between 0° C. and 100° C., being about half the variation of German silver. Its specific resistance ranged in different specimens from 2.9×10^4 to 3.7×10^4 C.G.S.

235. For the temperature-coefficient of a mercury-in-

* Benoît, "Etudes expérimentales sur la Résistance électrique sous l'Influence de la Température." Paris, 1873.

glass standard, the following mean values have been found :—

Rayleigh,	·000 861	between 0° and 12°
Glazebrook,	834	„ „ 5
„	861	„ „ 10
„	879	„ „ 15
Lorenz,	901	„ „ 27·3

Assuming the resistance to vary as $1 + \alpha t + \beta t^2$, the following values of α and β have been adopted :—

Strecker,	·000 900	·000 000 45
Siemens and Halske, ...	8523	001 356
Benoit,	8649	001 12
Guillaume (1890),	8769	001 012

Strecker's values are generally accepted in Germany. Guillaume, after allowing for the expansion of the hard glass employed, deduced for the specific resistance of mercury the coefficients

$$\alpha = \cdot 000\ 8841 \quad \beta = \cdot 000\ 001\ 025.$$

Resistances of Liquids.

236. The following tables of specific resistances of solutions are from the experiments of Ewing and Macgregor (Trans. Roy. Soc., Edin., xxvii. 1873) :—

Solutions at 10° C.		Specific Resistance.
Sulphate of Zinc, saturated,		$3\cdot37 \times 10^{10}$
„ „ minimum,		2·83 „
Sulphate of Copper, saturated,		2·93 „
Sulphate of Potash,		1·66 „
Bichromate of Potash,		2·96 „

The following table is for solutions of sulphate of copper of various strengths. The first column gives the ratio by weight of the crystals to the water in which they are dissolved :—

Strength.	Density at 10° C.	Specific Resistance.	Strength.	Density at 10°.	Specific Resistance.
1 to 40	1·0167	$16\cdot44 \times 10^{10}$	1 to 4·146	1·1386	$3\cdot5 \times 10^{10}$
30	1·0216	13·48 „	4	1·1432	3·41 „
20	1·0318	9·87 „	3·297	1·1679	3·17 „
10	1·0622	5·90 „	3	1·1823	3·06 „
7	1·0858	4·73 „	2·597 }	1·2051	2·93 „
5	1·1174	3·81 „	saturated }		

The following table is for solutions of sulphate of zinc:—

Strength.	Density.	Specific Resistance.	Strength.	Density.	Specific Resistance.
1 to 40	1·0140	$18·29 \times 10^{10}$	1 to 3	1·1582	$3·37 \times 10^{10}$
20	1·0278	11·11 „	2	1·2186	3·03 „
10	1·0540	6·38 „	1·5	1·270	2·85 „
7	1·0760	5·08 „	1	1·3530	3·10 „
5	1·1019	4·21 „	·752 } saturated }	1·4220	3·37 „

The following table for dilute sulphuric acid is from Becker's experiments, as quoted by Jamin and Bouty, tom. iv. p. 111:—

Density.	Specific Resistance.			
	At 0°.	At 8°.	At 16°.	At 24°.
1·10	$1·37 \times 10^{10}$	$1·04 \times 10^{10}$	$·845 \times 10^{10}$	$·737 \times 10^{10}$
1·20	1·33 „	·926 „	·666 „	·486 „
1·25	1·31 „	·896 „	·624 „	·434 „
1·30	1·36 „	·94 „	·662 „	·472 „
1·40	1·69 „	1·30 „	1·05 „	·896 „
1·50	2·74 „	2·13 „	1·72 „	1·52 „
1·60	4·82 „	3·62 „	2·75 „	2·21 „
1·70	9·41 „	6·25 „	4·23 „	3·07 „

Resistance of Carbons.

237. The specific resistance of Carré's electric-light carbons at 20° C. is stated to be

$$3·927 \times 10^6 \text{ C.G.S.},$$

whence it follows that the resistance of a cylinder 1 metre long and 1 centimetre in diameter is just half an ohm.

The specific resistance of Gaudin's carbons is about $8·5 \times 10^6$

„ „ retort carbon „ $6·7 \times 10^7$

„ „ graphite from $2·4 \times 10^6$ to $4·2 \times 10^7$

The resistance of carbon diminishes as the temperature increases, the diminution from 0° to 100° C. being $\frac{1}{19}$ for Carré's and $\frac{1}{24}$ for Gaudin's. The resistance of an incan-

descent lamp when heated as in actual use is about half its resistance cold.

Resistance of the Electric Arc.

238. The difference of potentials between the two carbons of an arc lamp has been found by Ayrton and Perry (Phil. Mag., May, 1883) to be practically independent of the strength of the current, when the distance between them is kept constant. It was scarcely altered by tripling the strength of the current. The apparent resistance of the arc (including the effect of reverse electromotive force) is therefore inversely as the current. The difference of potentials was about 30 volts when the current was from 6 to 12 amperes.

239. The following approximate determinations of the resistance of water and ice at different temperatures are contained in a paper by Professors Ayrton and Perry, dated March, 1877 (Proc. Phys. Soc., London, vol. ii., p. 178):—

	Temp. Cent.		Specific Resistance.
	- 12·4	$2\cdot240 \times 10^{18}$
	- 6·2	1·023 „
	- 5·02	$9\cdot486 \times 10^{17}$
	- 3·5	6·428 „
	- 3·0	5·693 „
	- 2·46	4·844 „
	- 1·5	3·876 „
	- 0·2	2·840 „
	+ 0·75	1·188 „
about	+ 2·2	$2\cdot48 \times 10^{16}$
	+ 4·0	$9\cdot1 \times 10^{15}$
	+ 7·75	$5\cdot4 \times 10^{14}$
	+ 11·02	3·4 „

The values in the original are given in megohms, and we have assumed the megohm = 10^{15} C.G.S. units.

According to F. Kohlrausch (Wied. Ann., xxiv. p. 48, 1885) the resistance at 18° C. of water purified by distillation in vacuo is 4×10^{10} times that of mercury. This makes its specific resistance

$$3.76 \times 10^{15}.$$

240. The specific resistance of glass of various kinds at various temperatures has been determined by Mr. Thomas Gray (Proc. Roy. Soc., Jan. 12, 1882). The following are specimens of the results:—

Bohemian Glass Tubing, density 2.43.

At 60°	6.05×10^{22}	At 160°	2.4×10^{19}
100	2×10^{21}	174	8.7×10^{18}
130	2×10^{20}		

Thomson's Electrometer Jar (flint glass), density 3.172.

At 100°	2.06×10^{23}	At 160°	2.45×10^{21}
120	4.68×10^{22}	180	5.6×10^{20}
140	1.06 ,,	200	1.2 ,,

The following are all at 60° C.:—

Bohemian Beaker,	4.25×10^{22}	density 2.427
„ „	7.15 „	2.587
Florence Flask,	4.69×10^{20}	2.523
Test Tube,	1.44 „	2.435
„ „	3.50 „	2.44
Flint Glass Tube,	3.89×10^{22}	2.753
Thomson's Electro- meter Jar (flint glass),	1.02×10^{24}	3.172

241. The following approximate values of the specific resistance of insulators after several minutes' electrification are given in a paper by Professors Ayrton and

Perry (Proc. Royal Society, March 21, 1878), "On the Viscosity of Dielectrics":—

	Specific Resistance.	Temperature. Centigrade.	Authority.
Mica,	8.4×10^{22}	20	Ayrton and Perry.
Gutta-Percha,	4.5×10^{23}	24	{ Standard adopted by Latimer Clark.
Shellac,	9.0×10^{24}	28	
Hooper's Material,	1.5×10^{25}	24	Recent cable tests.
Ebonite,	2.8×10^{25}	46	Ayrton and Perry.
Paraffin,	3.4×10^{25}	46	,,
Glass,	{ Not yet measured with accuracy, but greater than any of the above.		
Air,	Practically infinite.		

242. *Particulars of Board of Trade Standard Gauge of Wires (Imperial Gauge) Nos. 4 to 20.*

No.	Diameter.		Sectional area. Sq. inches.	Resistance in ohms of 1 metre length pure copper at 0° C.	
	Milli-metres.	Thou-sandths of inch.		Annealed.	Hard-drawn.
4	5.89	232	.04227	.0005929	.0006065
5	5.38	212	.03530	7107	7269
6	4.88	192	.02895	8638	8835
7	4.47	176	.02433	.001029	.001053
8	4.06	160	.02011	1248	1276
9	3.66	144	.01629	1536	1571
10	3.25	128	.01287	1948	1992
11	2.95	116	.01057	2364	2418
12	2.64	104	.008494	2951	3019
13	2.34	92	.006647	3757	3842
14	2.03	80	.005026	4992	5106
15	1.83	72	.004070	6142	6283
16	1.63	64	.003216	7742	7919
17	1.42	56	.002463	.01020	.01043
18	1.22	48	.001809	.01382	.01414
19	1.016	40	.001256	.01993	.02038
20	0.914	36	.000917	.02462	.02518

The heat generated per second in 1 metre length of pure copper wire at 0° C. is $\cdot 0048 \left(\frac{C}{D} \right)^2$ gm. deg., and at 40° C. is $\cdot 0055 \left(\frac{C}{D} \right)^2$ gm. deg., C denoting the current in amperes, and D the diameter in millimetres.

243. *Resistance of 1 metre length of Wires of Imperial Gauge at 0° C.* (For copper see preceding table.)

No.	Iron annealed.	German Silver, either annealed or hard-drawn.	Platinum annealed.	Silver, annealed.
4	$\cdot 003606$	$\cdot 007768$	$\cdot 003361$	$\cdot 0005583$
5	4322	9311	4028	6692
6	5253	$\cdot 01132$	4896	8184
7	6261	$\cdot 01349$	5836	9694
8	7590	$\cdot 01635$	7674	$\cdot 001175$
9	9339	$\cdot 02012$	8705	1446
10	$\cdot 01184$	$\cdot 02552$	$\cdot 01104$	1834
11	$\cdot 01438$	$\cdot 03096$	$\cdot 01340$	2226
12	$\cdot 01795$	$\cdot 03867$	$\cdot 01673$	2779
13	$\cdot 02285$	$\cdot 04922$	$\cdot 02129$	3538
14	$\cdot 03036$	$\cdot 06540$	$\cdot 02829$	4700
15	$\cdot 03736$	$\cdot 08047$	$\cdot 03482$	5784
16	$\cdot 04708$	$\cdot 1014$	$\cdot 04388$	7290
17	$\cdot 06204$	$\cdot 1336$	$\cdot 05782$	9606
18	$\cdot 08405$	$\cdot 1811$	$\cdot 07834$	$\cdot 01301$
19	$\cdot 1212$	$\cdot 2611$	$\cdot 1130$	$\cdot 01876$
20	$\cdot 1498$	$\cdot 3226$	$\cdot 1396$	$\cdot 02319$

Electromotive Force.

244. The electromotive force of a Daniell's cell was found by Sir W. Thomson (p. 246 of Papers on Electricity and Magnetism) to be

$\cdot 00374$ electrostatic unit,

from observation of the attraction between two parallel discs connected with the opposite poles of a Daniell's battery. As 1 electrostatic unit is 3×10^{10} electromagnetic units, this is $\cdot 00374 \times 3 \times 10^{10} = 1\cdot 122 \times 10^8$ electromagnetic units, or 1·122 volt.

According to Latimer Clark's experimental determinations communicated to the Society of Telegraph Engineers in January, 1873, the electromotive force of a Daniell's cell with pure metals and saturated solutions, at 64° F., is 1·105 volt., and the electromotive force of a Grove's cell 1·97 volt. These must be diminished by 1 per cent. because they were deduced from the assumption that the B.A. unit of resistance was correct. They will thus be reduced to 1·094 and 1·95 volts.

According to the determination of F. Kohlrausch (Pogg. Ann., vol. cxli. [1870], and *Ergänz.*, vol. vi. [1874], p. 35) the electromotive force of a Daniell's cell is $1\cdot 138 \times 10^8$, and that of a Grove's cell $1\cdot 942 \times 10^8$. These must be diminished by 3 per cent., because they were deduced from the value $\cdot 9717 \times 10^9$ for Siemens' unit which is 3 per cent. too great. They will thus be reduced to 1·104 and 1·884 volts.

H. S. Carhart (*Amer. Jour. Sci. Art.*, Nov. 1884) found the following different values for the electro-motive force of a Daniell's cell according to the strength of the zinc sulphate solution :—

Per Cent. of ZnSO_4 .	Electromotive force in volts.	Per Cent.	Electromotive force.
1	1·125	10	1·118
3	1·133	15	1·115
5	1·142	20	1·111
7½	1·120	25	1·111

He finds by the same method the electromotive force of Latimer Clark's standard cell to be 1.434 volt.

Lord Rayleigh (Phil. Trans., June, 1884, p. 452) determined the electromotive force of a Clark cell at 15° C. to be

$$1.435 \text{ volt.}$$

In a supplementary paper (Jan. 21, 1886) he gave the general result for any temperature t° ,

$$1.435\{1 - 0.00077(t - 15)\},$$

together with full particulars as to the precautions necessary for securing constancy.

245. Professors Ayrton and Perry have made determinations of the electromotive forces called out by the contacts, two and two, of a great number of substances measured inductively in air. The method of experimenting is described in the Proceedings of the Royal Society for March 21, 1878. The following abstract of their latest results was specially prepared for this work by Professor Ayrton in January, 1879 :—

	Carbon.	Copper.	Iron.	Lead.	Platinum.	Tin.	Zinc.	Amalgamated Zinc.	Brass.	Average Temperature at the time of experimenting, 18° C. about.
Carbon,	0	·370	·485*	·858	·113	·795	1·096	1·208*	·414*	
Copper,	— ·370	0	·146	·542	— ·238	·456	·750	·894	·087	
Iron,	— ·485*	— ·146	0	·401*	— ·369	·313*	·600*	·744*	— ·064	
Lead,	— ·858	— ·542	— ·401*	0	— ·771	— ·099	·210	·357*	— ·472	
Platinum,	— ·113*	·238	·369	·771	0	·690	·981	1·125*	·287	
Tin,	— ·795*	— ·456	— ·313*	·099	— ·690	0	·281	·463	— ·372	
Zinc,	— 1·096	— ·750	— ·600*	— ·210	— ·981	— ·281	0	·144	— ·679	
Amalgamated Zinc,	— 1·208*	— ·894	— ·744*	— ·357*	— 1·125*	— ·463	— ·144	0	— ·822*	
Brass,	— ·414	— ·087	·064	·472	— ·287	·372	·679	·822	0	

The numbers without an asterisk were obtained directly by experiment, those with an asterisk by calculation, using the well-known assumption that in a compound circuit of metals, all at the same temperature, there is no electromotive force. The numbers in a vertical column below the name of a substance are the differences of potential, in volts, between that substance and the substance in the same horizontal row as the number, the two substances being in contact. Thus, lead is positive to copper, the electromotive force of contact being 0·542 volts.

The metals were those of commerce, and therefore only commercially pure.

CONTACT DIFFERENCES OF POTENTIAL IN VOLTS.

		Carbon.	Copper.	Iron.	Lead.	Platinum.	Tin.
DILUTE SULPHURIC ACID.	SOLUTIONS.						
	Mercury,.....	·092	·308	·502	..	·156	..
	Distilled Water, {	·01 to 17 depending on carbon.	·269 to ·100	·148	·171	·285 to ·345	·177
	Alum, saturated at 16°·5 C.,..... {	..	·127	·653	·139	·246	·225
	Copper sulphate solution, specific gravity, 1·087 at 16°·6 C., {	..	·103
	Copper sulphate, saturated at 15° C.,..... {	..	·070
	Sea salt, specific gravity, 1·18 at 20°·5 C.,..... {	..	·475	·605	·267	·856	·334
	Sal-ammoniac, saturated at 15°·5 C.,... {	..	·396	·652	·189	·057	·364
	Zinc sulphate solution, specific gravity, 1·125 at 16°·9 C., {
	Zinc Sulphate, saturated at 15°·3 C.,... {
	1 Distilled water mixed with 3 zinc sulphate, saturated solution, {
	20 Distilled water, 1 strong sulphuric acid,..... {
	10 Distilled water, 1 strong sulphuric acid,..... {	about ·035
	5 Distilled water, 1 strong sulphuric acid,..... {
	1 Distilled water, 5 strong sulphuric acid,..... {	·3 to ·01	·120	..	·256
CONCENTRATED.	Sulphuric Acid, {	·85 to ·55 depending on carbon.	1·113	..	·720 to 1·252	1·600 to 1·300	..
	Nitric Acid,.....	·672	..
	Mercurous sulphate paste,..... {
	Distilled water, with a trace of sulphuric acid,..... {

The authors point out that in all these experiments the unknown electromotive forces of certain air contacts are included.

From these tables we find we can build up the electromotive forces of some well-known cells. For example, in a Daniell's cell there are four contact differences of potential to consider, and in a Grove's cell five, viz.:—

Daniell's Cell.

	Volts.
Copper and saturated copper sulphate,	+ 0·070
Saturated copper sulphate and saturated zinc sulphate,	- 0·095
Saturated zinc sulphate and zinc,	+ 0·430
Zinc and copper,	+ 0·750
	<hr/> 1·155

Grove's Cell.

Copper and platinum,	+ 0·238
Platinum and strong nitric acid,	+ 0·672
Strong nitric acid and very weak sulphuric acid,	+ 0·078
Very weak sulphuric acid and zinc,	+ 0·241
Zinc and copper,	+ 0·750
	<hr/> 1·979

Thermoelectricity.

246. The electromotive force of a thermoelectric circuit is called *Thermoelectric force*. It is proportional *cæt. par.* to the number of couples. The thermoelectric force of a single couple is in the majority of cases equal to the product of two factors, one being the difference of temperature of the two junctions, and the other the difference of the *thermoelectric heights* of the two metals at a temperature midway between those of the junctions. The current through the hot junction is from the lower

to the higher metal when their heights are measured at the mean temperature.

Our convention as to sign (that is, as to *up* and *down* in speaking of thermoelectric height) is the same as that adopted by Prof. Tait, and is opposite to that adopted in the first edition of this work. We have adopted it because it leads to the rule (for the Peltier and Thomson effects) that a current running down generates heat, and a current running up consumes heat.

The following table of thermoelectric heights relative to lead can be employed when the mean temperature of the two junctions does not differ much from 19° or 20° C. It is taken from Jenkin's *Electricity and Magnetism*, p. 176, where it is described as being compiled from Matthiessen's experiments. We have reversed the signs to suit the above convention, and have multiplied by 100 to reduce from microvolts to C.G.S. units.

Thermoelectric Heights at about 20° C.

Bismuth, pressed com- mercial wire,.....	- 9700	Antimony, pressed wire +	280
Bismuth, pure pressed wire,	- 8900	Silver, pure hard,	+ 300
Bismuth, crystal, axial, -	6500	Zinc, pure pressed,	+ 370
„ equatorial, ... -	4500	Copper, galvano-plas- tically precipitated, }	+ 380
Cobalt,	- 2200	Antimony, pressed commercial wire, ... }	+ 600
German Silver,	- 1175	Arsenic,	+ 1356
Quicksilver,	- 41·8	Iron, pianoforte wire, ...	+ 1750
Lead,	0	Antimony, axial,	+ 2260
Tin,	+ 10	„ equatorial, +	2640
Copper of Commerce, ...	+ 10	Phosphorus, red,	+ 2970
Platinum,	+ 90	Tellurium,	+ 50200
Gold,	+ 120	Selenium,	+ 80700

247. The following table is based upon Professor Tait's thermoelectric diagram (*Trans. Roy. Soc., Edin., vol.*

xxvii. 1873) joined with the assumption that a Grove's cell has electromotive force 1.97×10^8 :—

	Thermoelectric Heights at t° C. in C.G.S. units.	
Iron,	+ 1734	— 4.87 t
Steel,	+ 1139	— 3.28 t
Alloy, believed to be Platinum Iridium, +	839	at all temperatures.
Alloy, Platinum 95 ; Iridium 5,	+ 622	— .55 t
„ „ 90 ; „ 10,	+ 596	— 1.34 t
„ „ 85 ; „ 15,	+ 709	— .63 t
„ „ 85 ; „ 15,	+ 577	at all temperatures.
Soft Platinum,	— 61	— 1.10 t
Alloy, platinum and nickel,	+ 544	— 1.10 t
Hard Platinum,	+ 260	— .75 t
Magnesium,	+ 244	— .95 t
German Silver,	— 1207	— 5.12 t
Cadmium,	+ 266	+ 4.29 t
Zinc,	+ 234	+ 2.40 t
Silver,	+ 214	+ 1.50 t
Gold,	+ 283	+ 1.02 t
Copper,	+ 136	+ .95 t
Lead,	0	
Tin,	— 43	+ .55 t
Aluminium,	— 77	+ .39 t
Palladium,	— 625	— 3.59 t
Nickel to 175° C.,	— 2204	— 5.12 t
„ 250° to 310° C.,	— 8449	+ 24.1 t
„ from 340° C.,	— 307	— 5.12 t

The lower limit of temperature for the table is -18° C. for all the metals in the list. The upper limit is 416° C., with the following exceptions :—Cadmium, 258° C.; Zinc, 373° C.; German Silver, 175° C.

Ex. 1. Required the electromotive force of a copper-iron couple, the temperatures of the junctions being 0° C. and 100° C.

We have, for iron, $+ 1734 - 4.87 t$;

„ copper, $+ 136 + .95 t$;

„ iron above copper, $1598 - 5.82 t$.

The electromotive force per degree is

$$1598 - 5.82 \times 50 = 1307,$$

and the electromotive force of the couple is

$$1307 (100 - 0) = 130,700,$$

tending from copper to iron through the hot junction.

By the *neutral point* of two metals is meant the temperature at which their thermoelectric heights are equal.

Ex. 2. To find the neutral point of copper and iron we have

$$1598 - 5.82 t = 0, \quad t = 275 ;$$

that is, the neutral point is 275° C. When the mean of the temperatures of the junctions is below this point, the current through the warmer junction is from copper to iron. The current ceases as the mean temperature attains the neutral point, and is reversed in passing it.

Ex. 3. F. Kohlrausch (Pogg. Ann. Ergänz., vol. vi. p. 35, 1874) states that, according to his determination, the electromotive force of a couple of iron and German silver is 24×10^5 millimetre-milligramme-second units for 1° of difference of temperatures of the junctions, at moderate temperatures. Compare this result with the above Table at mean temperature 100° .

The dimensions of electromotive force are $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$; hence the C.G.S. value of Kohlrausch's unit is $10^{-\frac{3}{2}} 10^{-3} = 10^{-8}$, giving 2400 as the electromotive force per degree of difference.

From the above table we have

$$\text{Iron above German silver, } 2941 + .25 t,$$

which, for $t = 100$, gives 2966 as the electromotive force per degree of difference.

Peltier and Thomson Effects.

248. When a current is sent through a circuit composed of different metals, it produces in general three distinct thermal effects.

1. A generation of heat to the amount per second of C^2R ergs, C denoting the current, and R the resistance.

2. A generation of heat or cold at the junctions. This is called the Peltier effect, and its amount per second in ergs at any one junction can be computed by multiplying the difference of thermoelectric heights at this junction by $t + 273$ and by the current, t denoting the centigrade temperature of the junction. If the current flows down (that is from greater to less thermoelectric height) the effect is a warming; if it flows up, the effect is a cooling.

Ex. 4. Let a unit current (or a current of 10 amperes) flow through a junction of copper and iron at 100°C .

The thermoelectric heights at 100°C . are

Iron,	1247
Copper,	231
Iron above copper,	1016

Multiplying 1016 by 373, we have about 379,000 ergs, or $\frac{1}{111}$ of a gramme-degree, as the Peltier effect per second.

Heat of this amount will be generated if the current is from iron to copper, and will be destroyed if the current is from copper to iron.

3. A generation of heat or cold in portions of the circuit consisting of a single metal in which the temperature

varies from point to point. This is called the Thomson effect. Its amount per second, for any such portion of the circuit, is the difference of the thermoelectric heights of the two ends of the portion, multiplied by $273 + t$, where t denotes the half sum of the centigrade temperatures of the ends, and by the strength of the current.

The Thomson effect, like the Peltier effect, is reversed by reversing the current, and follows the same rule that *heat is generated* when the current is from *greater to less thermoelectric height*.

Experiment shows that the Thomson effect is insensible in the case of lead; hence the thermoelectric height of lead must be sensibly the same at all temperatures. It is for this reason that lead is adopted, by common consent, as the zero from which thermoelectric heights are to be reckoned.

Ex. 5. In an iron wire with ends at 0° C. and 100° C., the cold end is the higher (thermoelectrically) by 4.87×100 —that is, by 487. Multiplying this difference by $273 + \frac{1}{2}(0 + 100)$ or 323, we have 157300 as the Thomson effect per second for unit current. This amount of heat (in ergs) is generated in the iron when the current through it is from the cold to the hot end, and is destroyed when the current is from hot to cold.

Ex. 6. In a copper wire with ends at 0° C. and 100° C., the hot end is the higher by $.95 \times 100$ or 95. Multiplying this by 323, we have 30700 (ergs) as the Thomson effect per second per unit current. This amount of heat is generated in the copper when the current through it is from hot to cold, and destroyed when the current is from cold to hot.

The effect of a current from hot to cold is opposite in these two metals, because the coefficients of t in the expressions for their thermoelectric heights (§ 247) have opposite signs.

Relation between Thermoelectric Force and the Peltier and Thomson Effects.

249. The algebraic sum of the Peltier and Thomson effects (expressed in ergs) due to unit current for one second in a closed metallic circuit, is equal to the thermoelectric force of the circuit; and the direction of this thermoelectric force is the direction of a current round the circuit which would give an excess of destruction over generation of heat (so far as these two effects are concerned).

Ex. 7. In a copper-iron couple with junctions at 0°C. and 100°C. , suppose a unit current to circulate in such a direction as to pass from copper to iron through the hot junction, and from iron to copper through the cold junction.

The Peltier effect at the hot junction is a destruction of heat to the amount $1016 \times 373 = 379,000$ ergs.

The Peltier effect at the cold junction is a generation of heat to the amount $1598 \times 273 = 436,300$ ergs.

The Thomson effect in the iron is a destruction of heat to the amount $487 \times 323 = 157,300$ ergs.

The Thomson effect in the copper is a destruction of heat to the amount $95 \times 323 = 30,700$ ergs.

The total amount of destruction is 567,000, and of generation 436,300, giving upon the whole a destruction of 130,700 ergs. The electromotive force of the couple

is therefore 130,700, and tends in the direction of the current here supposed. This agrees with the calculation in Example 1.

Electrochemical Equivalents.

250. The quantity of a given metal deposited in an electrolytic cell or dissolved in a battery cell (when there is no "local action") depends on the quantity of electricity that passes, irrespective of the time occupied. Hence we can speak definitely of the quantity of the metal that is "equivalent to" a given quantity of electricity. By the *electrochemical equivalent* of a metal is meant the quantity of it that is equivalent to the unit quantity of electricity. In the C.G.S. system it is the number of grammes of the metal that are equivalent to the C.G.S. electromagnetic unit of electricity.

Special attention has been paid to the electrochemical equivalent of silver, as this metal affords special facilities for an accurate determination. The latest experiments of Lord Rayleigh and Kohlrausch agree in giving

$$0.1118$$

as the C.G.S. electrochemical equivalent of silver.*

The number of grammes of silver deposited by 1 ampere in one hour is

$$0.1118 \times \frac{1}{10} \times 3600 = 4.025.$$

251. The electrochemical equivalents of the most important of the elements are given in the following table. They are calculated from the chemical equivalents in the preceding column by simple proportion, taking as basis the above-named value for silver. Their reciprocals are

* Rayleigh's determination is 0.111794; Kohlrausch's, 0.11183; Mascart's, 0.11156. See Phil. Trans., 1884, pp. 439, 458.

the quantities of electricity required for depositing one gramme. The quantity of electricity required for depositing the number of grammes stated in the column "chemical equivalents" is the same for all the elements, namely, 9634 C.G.S. units.

Elements.	Atomic Weight.	Valency.	Chemical Equivalents.	Electro-chemical equivalents or grammes per unit of electricity.	Reciprocal or Electricity per gramme.
Electro-positive—					
Hydrogen,.....	1	1	1	·0001938	9634
Potassium,	39·03	1	39·03	·004051	246·9
Sodium,.....	23·00	1	23·00	·002387	418·9
Gold,.....	196·2	3	65·4	·006789	147·3
Silver,	107·7	1	107·7	·01118	89·45
Copper (cupric),	63·18	2	31·59	·003279	305·0
„ (cuprous),	„	1	63·18	·006558	152·5
Mercury (mercuric),...	199·8	2	99·9	·01037	96·43
„ (mercurous),	„	1	199·8	·02074	48·22
Tin (stannic),.....	117·4	4	29·35	·003046	328·3
„ (stannous),.....	„	2	58·7	·006093	164·1
Iron (ferric),.....	55·88	3	18·63	·001934	517·1
„ (ferrous),.....	„	2	27·94	·002900	344·8
Nickel,.....	58·6	2	29·3	·003042	328·7
Zinc,	64·88	2	32·44	·003367	297·0
Lead,	206·4	2	103·2	·01071	93·37
Aluminium,.....	27·04	3	9·01	·000935	1070
Electro-negative—					
Oxygen,.....	15·96	2	7·98	·0008283	1207
Chlorine,.....	35·37	1	35·37	·003671	272·4
Iodine,.....	126·54	1	126·54	·013134	76·14
Bromine,.....	79·76	1	79·76	·008279	120·8
Nitrogen,.....	14·01	3	4·67	·0004847	2063

To find the equivalent of 1 coulomb, divide the above electrochemical equivalents by 10.

To find the number of grammes deposited per hour by

1 ampere, multiply the above electrochemical equivalents by 360.

252. Let the "chemical equivalents" in the above table be taken as so many grammes: then, if we denote by H the amount of heat due to the whole chemical action which takes place in a battery cell during the consumption of one equivalent of zinc, the chemical energy which runs down, namely JH ergs, must be equal (if there is no wasteful local action) to the energy of the current produced. But this is the product of the quantity of electricity 9634 by the electromotive force of the cell. The electromotive force is therefore equal to $\frac{JH}{9634}$.

In the tables of heats of combination which are in use among chemists, the equivalent of hydrogen is taken as 2 grammes, and that of zinc as 64.88 or 65 grammes. The equivalent quantity of electricity will accordingly be 9634×2 , and the formula to be used for calculating the electromotive force of a cell will be $\frac{JH}{19268}$.

In applying this calculation to Daniell's and Grove's cells, we shall employ the following heats of combination, which are given on page 614 of Watts' Dictionary of Chemistry, vol. vii., and are based on Julius Thomsen's observations:—

Zn, O, SO ³ , Aq.,.....	108,462
Cu, O, SO ³ , Aq.,.....	54,225
N ² O ² , O ³ , Aq.,.....	72,940
N ² O ² , O, Aq.,.....	36,340

In Daniell's cell, zinc is dissolved and copper is set free; we have, accordingly,

$$H = 108,462 - 54,225 = 54,237.$$

In Grove's cell, zinc is dissolved and nitric acid is changed into nitrous acid. The thermal value of this latter change can be computed from the third and fourth data in the above list, as follows:—

72,940 is the thermal value of the action in which, by the oxidation of one equivalent of N^2O^2 and combination with water, two equivalents of NHO^3 (nitric acid) are produced.

36,340 is the thermal value of the action in which, by the oxidation of one equivalent of N^2O^2 and combination with water, two equivalents of NHO^2 (nitrous acid) are produced. The difference 36,600 is accordingly the thermal value of the conversion of two equivalents of nitrous into nitric acid, and 18,300 is the value for the conversion of one equivalent. In the present case the reverse changes take place. We have, therefore,

$$H = 108,462 - 18,300 = 90,162.$$

Taking J as 4.2×10^7 , the value of $\frac{JH}{19268}$ will be

$$1.182 \times 10^8 \text{ for Daniell's cell.}$$

$$1.965 \times 10^8 \text{ ,, Grove's ,,}$$

These are greater by from 2 to 8 per cent. than the direct determinations given in § 244.

Examples in Electricity.

1. Two conducting spheres, each of 1 centim. radius, are placed at a distance of r centims. from centre to centre, r being a large number; and each of them is charged with an electrostatic unit of positive electricity. With what force will they repel each other?

Since r is large, the charge may be assumed to be uniformly distributed over their surfaces, and the force will

be the same as if the charge of each were collected at its centre. The force will therefore be $\frac{1}{r^2}$ of a dyne.

2. Two conducting spheres, each of 1 centim. radius, placed as in the preceding question, are connected one with each pole of a Daniell's battery (the middle of the battery being to earth) by means of two very fine wires whose capacity may be neglected, so that the capacity of each sphere when thus connected is sensibly equal to unity. Of how many cells must the battery consist that the spheres may attract each other with a force of $\frac{1}{r^2}$ of a dyne, r being the distance between their centres in centims.? (See § 244.)

One sphere must be charged to potential 1 and the other to potential -1 . The number of cells required is

$$\frac{2}{\cdot 00374} = 535.$$

3. How many Daniell's cells would be required to produce a spark between two parallel conducting surfaces at a distance of $\cdot 019$ of a centim., and how many at a distance of $\cdot 0086$ of a centim.? (See §§ 226, 244.)

$$\text{Ans. } \frac{4\cdot 26}{\cdot 00374} = 1139; \quad \frac{2\cdot 30}{\cdot 00374} = 615.$$

4. Compare the capacity denoted by 1 farad with the capacity of the earth.

The capacity of the earth in static measure is equal to its radius, namely $6\cdot 37 \times 10^8$. Dividing by v^2 to reduce to magnetic measure, we have $\cdot 71 \times 10^{-12}$, which is 1 farad multiplied by $\cdot 71 \times 10^{-3}$, or is $\cdot 00071$ of a farad. A farad is therefore 1400 times the capacity of the earth.

5. Calculate the resistance of a cell consisting of a

plate of zinc, A square centims. in area, and a plate of copper of the same dimensions, separated by an acid solution of specific resistance 10^9 , the distance between the plates being 1 centim.

Ans. $\frac{10^9}{A}$, or $\frac{1}{A}$ of an ohm.

6. Find the heat developed in 10 minutes by the passage of a current from 10 Daniell's cells in series through a wire of resistance of 10^{10} (that is, 10 ohms), assuming the electromotive force of each cell to be 1.1×10^8 , and the resistance of each cell to be 10^9 .

Here we have

$$\text{Total electromotive force} = 1.1 \times 10^9.$$

$$\text{Resistance in battery} = 10^{10}.$$

$$\text{Resistance in wire} = 10^{10}.$$

$$\text{Current} = \frac{1.1 \times 10^9}{2 \times 10^{10}} = .55 \times 10^{-1} = .055.$$

$$\text{Heat developed in } \left. \begin{array}{l} \text{wire per second} \end{array} \right\} = \frac{(.055)^2 \times 10^{10}}{4.2 \times 10^7} = 7.2024.$$

Hence the heat developed in 10 minutes is 4321.4 gramme-degrees.

7. Find the electromotive force between the wheels on opposite sides of a railway carriage travelling at the rate of 30 miles an hour on a line of the ordinary gauge [4 feet $8\frac{1}{2}$ inches] due to cutting the lines of force of terrestrial magnetism, the vertical intensity being .438.

The electromotive force will be the product of the velocity of travelling, the distance between the rails, and the vertical intensity, that is,

$$(44.7 \times 30) (2.54 \times 56.5) (.438) = 84,300$$

electromagnetic units.

This is about $\frac{1}{1200}$ of a volt.

8. Find the electromotive force at the instant of passing the magnetic meridian, in a circular coil consisting of 300 turns of wire, revolving at the rate of 10 revolutions per second about a vertical diameter; the diameter of the coil being 30 centims., and the horizontal intensity of terrestrial magnetism being $\cdot 1794$, no other magnetic influence being supposed present, and self-induction being left out of account.

The numerical value of the lines of force which go through the coil when inclined at an angle θ to the meridian, is the horizontal intensity multiplied by the area of the coil and by $\sin \theta$; say $nH\pi a^2 \sin \theta$, where $H = \cdot 1794$, $a = 15$, and $n = 300$. The electromotive force at any instant is the rate at which this quantity increases or diminishes; that is, $nH\pi a^2 \cos \theta \cdot \omega$, if ω denote the angular velocity. At the instant of passing the meridian, $\cos \theta$ is 1, and the electromotive force is $nH\pi a^2 \omega$. With 10 revolutions per second the value of ω is $2\pi \times 10$.

Hence the electromotive force is

$$\cdot 1794 \times (3 \cdot 142)^2 \times 225 \times 20 \times 300 = 2 \cdot 39 \times 10^6.$$

This is about $\frac{1}{42}$ of a volt.

Electrodynamics.

253. Ampère's formula for the repulsion between two elements of currents, when expressed in electromagnetic units, is

$$\frac{cc' ds \cdot ds'}{r^2} (2 \sin a \sin a' \cos \theta - \cos a \cos a'),$$

where c, c' denote the strengths of the two currents ;
 ds, ds' the lengths of the two elements ;
 α, α' the angles which the elements make with
the line joining them ;
 r the length of this joining line ;
 θ the angle between the plane of r, ds , and
the plane of r, ds' .

For two parallel currents, one of which is of infinite length, and the other of length l , the formula gives, by integration, an attraction or repulsion

$$\frac{2l}{D} cc',$$

where D denotes the perpendicular distance between the currents.

Example.

Find the attraction between two parallel wires a metre long and a centim. apart when a current of $\frac{1}{10}$ is passing through each.

Here the attraction will be sensibly the same as if one of the wires were indefinitely increased in length, and will be

$$\frac{200}{1} \left(\frac{1}{10} \right)^2 = 2 ;$$

that is, each wire will be attracted or repelled with a force of 2 dynes, according as the directions of the currents are the same or opposite.

Coefficients of Self-Induction and of Mutual Induction.

254. When a steady current C flows through a coil of resistance R , the difference of potential between the ends of the coil is CR . When the current, instead of being

steady, is increasing at the rate dC/dt , the difference of potentials between the ends is

$$CR + L \frac{dC}{dt},$$

L being a constant called the self-induction (or coefficient of self-induction) of the coil. This is on the supposition that no current except that in the coil itself exerts an inductive influence on the coil. LdC/dt may be described as the reverse e. m. f. due to the fact that the current is increasing.

When two neighbouring coils influence each other, the reverse e. m. f. in the first coil is

$$L_1 dC_1/dt + M dC_2/dt,$$

and the reverse e. m. f. in the second coil is

$$L_2 dC_2/dt + M dC_1/dt,$$

L_1 being the self-induction of the first coil, L_2 that of the second, and M the mutual induction of the two coils.

Since the two terms of the expression $CR + L dC/dt$ must be of the same dimensions, L has the dimensions Rt . In the electromagnetic system, R is a velocity, hence L has the dimensions of length. In C.G.S. units, R is in centims. per second, and L and M are in centims. In the "practical" system, R is in ohms, t is in seconds, and the unit for L and M has often been called the *secohm*, a name compounded of *ohm* and *second*. The official name recently adopted by an International Congress at Paris, and by the Electrical Standards Committee of the British Association, is the *quadrant*, an abbreviation of the name "earth-quadrant," the ohm being a velocity of an earth-quadrant per second. The accurate value is 10^9 centims.

255. The same authorities have introduced some new names in connection with alternating currents. Let E denote the difference of potential between the two ends of the armature coil, and let its value in terms of the time t be

$$E = E_0 \sin \omega t,$$

E_0 and ω being constants.

If R denote the resistance of the armature, the current would be

$$C = \frac{E_0}{R} \sin \omega t$$

if there were no self-induction. Let L be the self-induction, then the value of C is

$$C = \frac{E_0}{\sqrt{(R^2 + L^2\omega^2)}} \sin(\omega t - \beta),$$

β being a constant such that $\tan \beta = \frac{L\omega}{R}$. The quantity

$\sqrt{(R^2 + L^2\omega^2)}$ is called the *impedance* (the accent to be on the second syllable). This name is adopted by the Standards Committee in preference to *résistance apparente* which was recommended by the Paris Congress.

Both authorities agree in giving the name *effective current* to the quantity whose square is equal to the mean value of the square of C . It must be distinguished from the *mean current*, which, with the above value of C , would be zero.

256. To investigate the magnitudes of units of length, mass, and time which will fulfil the three following conditions:—

1. The acceleration due to the attraction of unit mass at unit distance shall be unity.

2. The electrostatic units shall be equal to the electromagnetic units.

3. The density of water at 4° C. shall be unity.

Let the 3 units required be equal respectively to L centims., M grammes, and T seconds.

We have in C.G.S. measure, for the acceleration due to attraction (§ 86),

$$\text{acceleration} = C \frac{\text{mass}}{(\text{distance})^2}, \text{ where } C = 6.48 \times 10^{-8};$$

and in the new system we are to have

$$\text{acceleration} = \frac{\text{mass}}{(\text{distance})^2}.$$

Hence, by division,

$$\frac{\text{acceleration in C.G.S. units}}{\text{acceleration in new units}} = C \frac{\text{mass in C.G.S. units}}{\text{mass in new units}} \cdot \frac{(\text{distance in new units})^2}{(\text{distance in C.G.S. units})^2};$$

$$\text{that is, } \frac{L}{T^2} = C \frac{M}{L^2}.$$

This equation expresses the first of the three conditions.

The equation $\frac{L}{T} = v$ expresses the second, v denoting 3×10^{10} .

The equation $M = L^3$ expresses the third.

Substituting L^3 for M in the first equation, we find

$$T = \sqrt{\frac{1}{C}}. \text{ Hence, from the second equation,}$$

$$L = v \sqrt{\frac{1}{C}};$$

and from the third,

$$M = \left(v \sqrt{\frac{1}{C}} \right)^3.$$

Introducing the actual values of C and v , we have approximately

$$T = 3928, L = 1.178 \times 10^{14}, M = 1.63 \times 10^{42};$$

that is to say,

The new unit of time will be about $1^h 5\frac{1}{2}^m$;

The new unit of length will be about 118 thousand earth quadrants;

The new unit of mass will be about 2.66×10^{14} times the earth's mass.

Modern Views on Electrical and Magnetic Dimensions.

257. Maxwell has pointed out (Elec. and Mag. § 622, 2nd edition) certain relations which must exist between the dimensions of various electrical and magnetic quantities in any consistent system, and has shown (§ 623) that the dimensions of any electrical or magnetic quantity can be definitely expressed—

(1) In terms of mass, length, time, and quantity of electricity;

(2) In terms of mass, length, time, and quantity of magnetism;

the dimensions thus obtained being the same for all systems.

Rücker (Phil. Mag., Feb. 1889) has pointed out, as an algebraic deduction from equations given by Maxwell, that the dimensions of all electrical and magnetic quantities can be definitely expressed—

(1) In terms of mass, length, time, and specific inductive capacity;

(2) In terms of mass, length, time, and magnetic permeability;

and maintains, in common with several of the leading exponents of Maxwell's views, that *specific inductive capacity* and *magnetic permeability*, which are usually regarded as mere numerical quantities, ought to be regarded as quantities of unknown dimensions. The numerical ratios usually understood by these terms must then be regarded not as absolute but merely as relative values, all of which are to be multiplied by the values for vacuum, which are at present unknown, and which depend, in some way not at present known, on the units of length, mass, and time employed.

The following are examples of the dimensions thus obtained, K denoting specific inductive capacity, and μ magnetic permeability.

	In terms of K.	In terms of μ .
{ Quantity of electricity,	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1} K^{\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{1}{2}} \mu^{-\frac{1}{2}}$
{ Strength of pole,	$M^{\frac{1}{2}} L^{\frac{1}{2}} K^{-\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1} \mu^{\frac{1}{2}}$
{ Electric force at a point, ...	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1} K^{-\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-2} \mu^{\frac{1}{2}}$
{ Magnetic ,, ,, ...	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-2} K^{\frac{1}{2}}$	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1} \mu^{-\frac{1}{2}}$
{ Electric potential,	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} K^{-\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2} \mu^{\frac{1}{2}}$
{ Magnetic ,,	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2} K^{\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} \mu^{-\frac{1}{2}}$
Intensity of magnetization,	$M^{\frac{1}{2}} L^{-\frac{3}{2}} K^{-\frac{1}{2}}$	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1} \mu^{\frac{1}{2}}$
Electrical resistance,	$L^{-1} T K^{-1}$	$L T^{-1} \mu$
,, capacity,	LK	$L^{-1} T^2 \mu^{-1}$

258. In every case the ratio of the dimensions in terms of K to the dimensions in terms of μ is a power of $K^{\frac{1}{2}} \mu^{\frac{1}{2}} L T^{-1}$.

If electrical and magnetic actions are to be regarded as manifestations of the ordinary laws of dynamics applied

to motions which our present knowledge does not enable us to specify, then every electrical or magnetic quantity has definite dimensions in terms of mass, length, and time; hence the expression $K^{\frac{1}{2}}\mu^{\frac{1}{2}}LT^{-1}$ must be of dimensions $M^0L^0T^0$, in other words $K^{\frac{1}{2}}\mu^{\frac{1}{2}}$ must be the reciprocal of a velocity.

APPENDIX.

First Report of the Committee for the Selection and Nomenclature of Dynamical and Electrical Units, the Committee consisting of
 SIR W. THOMSON, F.R.S., PROFESSOR G. C. FOSTER, F.R.S.
 PROFESSOR J. C. MAXWELL, F.R.S., MR. G. J. STONEY, F.R.S.,*
 PROFESSOR FLEEMING JENKIN, F.R.S., DR. SIEMENS, F.R.S.,
 MR. F. J. BRAMWELL, F.R.S., and PROFESSOR EVERETT
 (Reporter). [1873.]

WE consider that the most urgent portion of the task intrusted to us is that which concerns the selection and nomenclature of units of force and energy; and under this head we are prepared to offer a definite recommendation.

A more extensive and difficult part of our duty is the selection and nomenclature of electrical and magnetic units. Under this head we are prepared with a definite recommendation as regards selection, but with only an interim recommendation as regards nomenclature.

Up to the present time it has been necessary for every person who wishes to specify a magnitude in what is called "absolute" measure, to mention the three fundamental units of mass, length, and time which he has chosen as the basis of his system. This necessity will be obviated if one definite selection of three fundamental units be made once for all, and accepted by the general consent of scientific men. We are strongly of opinion that such a selection ought at once to be made, and to be so made that there will be no subsequent necessity for amending it.

* Mr. Stoney objected to the selection of the centimetre as the unit of length.

We think that, in the selection of each kind of derived unit, all arbitrary multiplications and divisions by powers of ten, or other factors, must be rigorously avoided, and the whole system of fundamental units of force, work, electrostatic, and electromagnetic elements must be fixed at one common level—that level, namely, which is determined by direct derivation from the three fundamental units once for all selected.

The carrying out of this resolution involves the adoption of some units which are excessively large or excessively small in comparison with the magnitudes which occur in practice; but a remedy for this inconvenience is provided by a method of denoting decimal multiples and sub-multiples, which has already been extensively adopted, and which we desire to recommend for general use.

On the initial question of the particular units of mass, length, and time to be recommended as the basis of the whole system, a protracted discussion has been carried on, the principal point discussed being the claims of the gramme, the *metre*, and the second, as against the gramme, the *centimetre*, and the second,—the former combination having an advantage as regards the simplicity of the name *metre*, while the latter combination has the advantage of making the unit of mass practically identical with the mass of unit-volume of water—in other words, of making the value of the density of water practically equal to unity. We are now all but unanimous in regarding this latter element of simplicity as the more important of the two; and in support of this view we desire to quote the authority of Sir W. Thomson, who has for a long time insisted very strongly upon the necessity of employing units which conform to this condition.

We accordingly recommend the general adoption of the *Centimetre*, the *Gramme*, and the *Second* as the three fundamental units; and until such time as special names shall be appropriated to the units of electrical and magnetic magnitude hence derived, we recommend that they be distinguished from “absolute” units otherwise derived, by the letters “C.G.S.” prefixed, these being the initial letters of the names of the three fundamental units.

Special names, if short and suitable, would, in the opinion of a majority of us, be better than the provisional designations “C.G.S.

unit of" Several lists of names have already been suggested; and attentive consideration will be given to any further suggestions which we may receive from persons interested in electrical nomenclature.

The "ohm," as represented by the original standard coil, is approximately 10^9 C.G.S. units of resistance; the "volt" is approximately 10^8 C.G.S. units of electromotive force; and the "farad" is approximately $\frac{1}{10^9}$ of the C.G.S. unit of capacity.

For the expression of high decimal multiples and sub-multiples, we recommend the system introduced by Mr. Stoney, a system which has already been extensively employed for electrical purposes. It consists in denoting the exponent of the power of 10, which serves as a multiplier, by an appended cardinal number, if the exponent be positive, and by a prefixed ordinal number if the exponent be negative.

Thus 10^9 grammes constitute a *gramme-nine*; $\frac{1}{10^9}$ of a gramme constitutes a *ninth-gramme*: the approximate length of a quadrant of one of the earth's meridians is a *metre-seven*, or a *centimetre-nine*.

For multiplication or division by a million, the prefixes *mega** and *micro* may conveniently be employed, according to the present custom of electricians. Thus the *megohm* is a million ohms, and the *microfarad* is the millionth part of a farad. The prefix *mega* is equivalent to the affix *six*. The prefix *micro* is equivalent to the prefix *sixth*.

The prefixes *kilo*, *hecto*, *deca*, *deci*, *centi*, *milli* can also be employed in their usual senses before all new names of units.

As regards the name to be given to the C.G.S. *unit of force*, we recommend that it be a derivative of the Greek *δύναμις*. The form *dynamy* appears to be the most satisfactory to etymologists. *Dynam* is equally intelligible, but awkward in sound to English ears. The shorter form, *dyne*, though not fashioned according to strict rules of etymology, will probably be generally preferred in this country. Bearing in mind that it is desirable to construct a

* Before a vowel, either *meg* or *megal*, as euphony may suggest, may be employed instead of *mega*.

system with a view to its becoming international, we think that the termination of the word should for the present be left an open question. But we would earnestly request that, whichever form of the word be employed, its meaning be strictly limited to the unit of force of the C.G.S. system—that is to say, *the force which, acting upon a gramme of matter for a second, generates a velocity of a centimetre per second.*

The C.G.S. *unit of work* is the work done by *this force working through a centimetre*; and we propose to denote it by some derivative of the Greek $\epsilon\rho\gamma\omega\nu$. The forms *ergon*, *ergal*, and *erg* have been suggested; but the second of these has been used in a different sense by Clausius. In this case also we propose, for the present, to leave the termination unsettled; and we request that the word *ergon*, or *erg*, be strictly limited to the C.G.S. unit of work, or what is, for purposes of measurement, equivalent to this, the C.G.S. *unit of energy*, energy being measured by the amount of work which it represents.

The C.G.S. *unit of power* is the power of doing work at the rate of *one erg per second*; and the power of an engine, under given conditions of working, can be specified in *ergs per second*.

For rough comparison with the vulgar (and variable) units based on terrestrial gravitation, the following statement will be useful:—

The *weight* of a *gramme*, at any part of the earth's surface, is about 980 *dynes*, or rather less than a *kilodyne*.

The *weight* of a *kilogramme* is rather less than a *megadyne*, being about 980,000 *dynes*.

Conversely, the *dyne* is about 1.02 times the *weight* of a *milligramme* at any part of the earth's surface; and the *megadyne* is about 1.02 times the *weight* of a *kilogramme*.

The *kilogrammetre* is rather less than the *ergon-eight*, being about 98 million *ergs*.

The *gramme-centimetre* is rather less than the *kilerg*, being about 980 *ergs*.

For exact comparison, the value of *g* (the acceleration of a body falling in vacuo) at the station considered must of course be known. In the above comparison it is taken as 980 C.G.S. units of acceleration.

One *horse-power* is about three-quarters of an *erg-ten* per second. More nearly, it is 7·46 *erg-nines* per second, and one *force-de-cheval* is 7·36 *erg-nines* per second.

The mechanical equivalent of one *gramme-degree* (Centigrade) of heat is 41·6 megalergs, or 41,600,000 *ergs*.

Second Report of the Committee for the Selection and Nomenclature of Dynamical and Electrical Units, the Committee consisting of PROFESSOR SIR W. THOMSON, F.R.S., PROFESSOR G. C. FOSTER, F.R.S., PROFESSOR J. CLERK MAXWELL, F.R.S., G. J. STONEY, F.R.S., PROFESSOR FLEEMING JENKIN, F.R.S., DR. C. W. SIEMENS, F.R.S., F. J. BRAMWELL, F.R.S., PROFESSOR W. G. ADAMS, F.R.S., PROFESSOR BALFOUR STEWART, F.R.S., *and* PROFESSOR EVERETT (Secretary). [1874.]

THE Committee on the Nomenclature of Dynamical and Electrical Units have circulated numerous copies of their last year's Report among scientific men both at home and abroad.

They believe, however, that, in order to render their recommendations fully available for science teaching and scientific work, a full and popular exposition of the whole subject of physical units is necessary, together with a collection of examples (tabular and otherwise) illustrating the application of systematic units to a variety of physical measurements. Students usually find peculiar difficulty in questions relating to units; and even the experienced scientific calculator is glad to have before him concrete examples with which to compare his own results, as a security against misapprehension or mistake.

Some members of the Committee have been preparing a small volume of illustrations of the C.G.S. system [Centimetre-Gramme-Second system] intended to meet this want.

[The first edition of the present work is the volume of illustrations here referred to.]

Resolutions adopted by the International Congress of Electricians at Paris at the sitting of September 22nd, 1881.

1. For electrical measurements, the fundamental units, the centimetre (for length), the gramme (for mass), and the second (for time), are adopted.

2. The ohm and the volt (for practical measures of resistance and electromotive force or potential) are to keep their existing definitions, 10^9 for the ohm, and 10^8 for the volt.

3. The ohm is to be represented by a column of mercury of a square millimetre section at the temperature of zero centigrade.

4. An International Commission is to be appointed to determine, for practical purposes, by fresh experiments, the length of a column of mercury of a square millimetre section which is to represent the ohm.

5. The current produced by a volt through an ohm is to be called an ampère.

6. The quantity of electricity given by an ampère in a second is to be called a coulomb.

7. The capacity defined by the condition that a coulomb charges it to the potential of a volt is to be called a farad.

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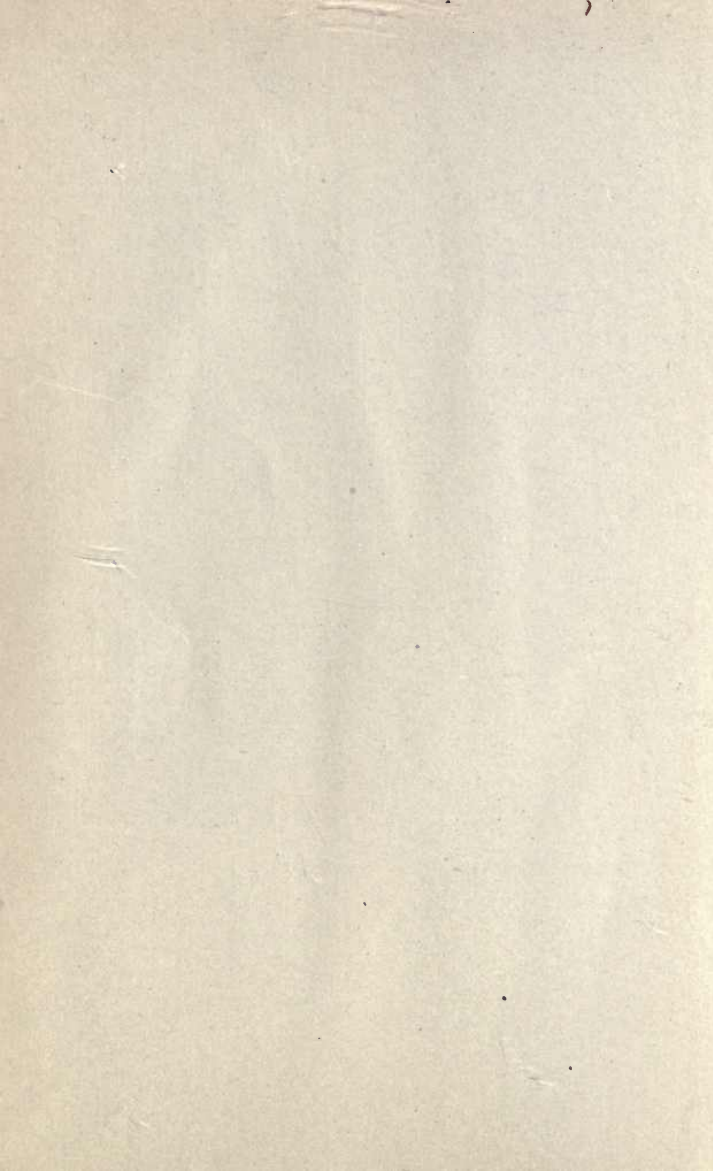
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